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Stochastic Modeling and Daily-Flows Generation at the North Fork of the Virgin River above Narrows Canyon, Zion National Park, Utah

by

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1.0 PURPOSE OF THE STUDY

Several considerations derived from the Virgin River adjudication study have indicated the need of having available a long-term series of natural flows for the upper reaches of the North Fork of the Virgin River (NFVR). More specifically, at a river site located upstream from the north boundary of Zion National Park, above the area known as Narrows Canyon.

To achieve this objective, a study was implemented, which consists of the stochastic modeling and synthetic data generation of daily flows. Historical data of daily streamflows compiled by the U.S. Geological Survey at several locations in the region were utilized. The stochastic modeling of daily flows was developed following current modeling procedures.

This report is organized in the following manner. Section 2.0 defines the general area of interest, the study site and specifies the hydrological data to be used. Section 3.0 carries out a preliminary data analysis, including extension of flow records and the estimation of long-term flow characteristics at the site of interest. Section 4.0 concentrates in the analysis of the time dependence structure of flows in the upper basin of the NFVR. Section 5.0 deals with the identification of the flow stochastic model. And finally Section 6.0, explains the generation process of daily flows and discusses some potential applications and limitations of the synthetically generated flows. To conclude this report, a list of references and appendices with numerical results are included.

The reader will find repeatedly references to a previous report by Diaz (1992), which provides background information about the general hydrologic conditions in the region. The report is entitled "Streamflow Characterization at Zion National Park, Utah", presented to the National Park Service in August 1992.

2.0 STUDY AREA

The general setting of Zion National Park within the upper basin of the Virgin River can be found in the previous report and several other listed references. The very brief description provided in this section concentrates on the North Fork of the Virgin River, which runs through Zion National Park in the North-South direction. The river area for the purpose of this study is located in the upper portion of the North Fork basin, outside the Park's boundaries, and upstream from the Narrows Canyon. More specifically, the study site is situated in the proximity of a potential dam site selected by the State of Utah, and depicted in Figure 1 as "Bullock Site" (approximate location). The cartographic location of the Bullock dam site is identified by Township 39S, Range 9W and Section 32,33. According to the Utah Division of Water Resources, the Bullock site is located in the Carmel formation immediately above the Navajo formation.

The NFVR subbasin encompasses a total of 123.61 square miles. The stream extends for approximately 38.8 miles, from the watershed divide in the Cascades Spring area at approximately elevation 9700 ft, to the lowest portion at the confluence with the East Fork of the Virgin River (EFVR), elevation 3,770 ft. Water accretion from Cascade Springs contributes much of the flow to the highest reaches of the North Fork. Deep snow packs and summer thunderstorms contribute the non-uniform component of the surface runoff, whereas discharge from springs and seeps basically comprise the base flow.

The flow data available, in the form of mean-daily values, has been collected and compiled by the U.S. Geological Survey. The three USGS gaging stations within the area of interest for this study are listed in Table 1. The location of these stations are also shown in Figure 1, referenced by their corresponding site number. Gaging stations at sites (8) and (9) are both located in the upper portions of the basin, in close proximity to each other, only 3.3 miles apart, and also close to the study site. For all practical purposes, this study assumes that flows registered at site (9), USGS station No. 09405450 - NFVR above Zion Narrows Canyon, are representative of natural flows conditions at the study site (Bullock site).

Table 1. List of Gaging Station Used in this Study

Site No.	USGS No.	Station Name and Location	Period of Record	Drainage Area (mi^2)	Elevation (ft)
(8)	09405420	North Fork Virgin River below Bullock Canyon	1975-1984	30.0	6420.
(9)	09405450	North Fork Virgin River above Zion Narrows	1979-1984	42.0	6000.
(12)	09405500	North Fork Virgin River near Springdale	1926-1991	344.0	3970.

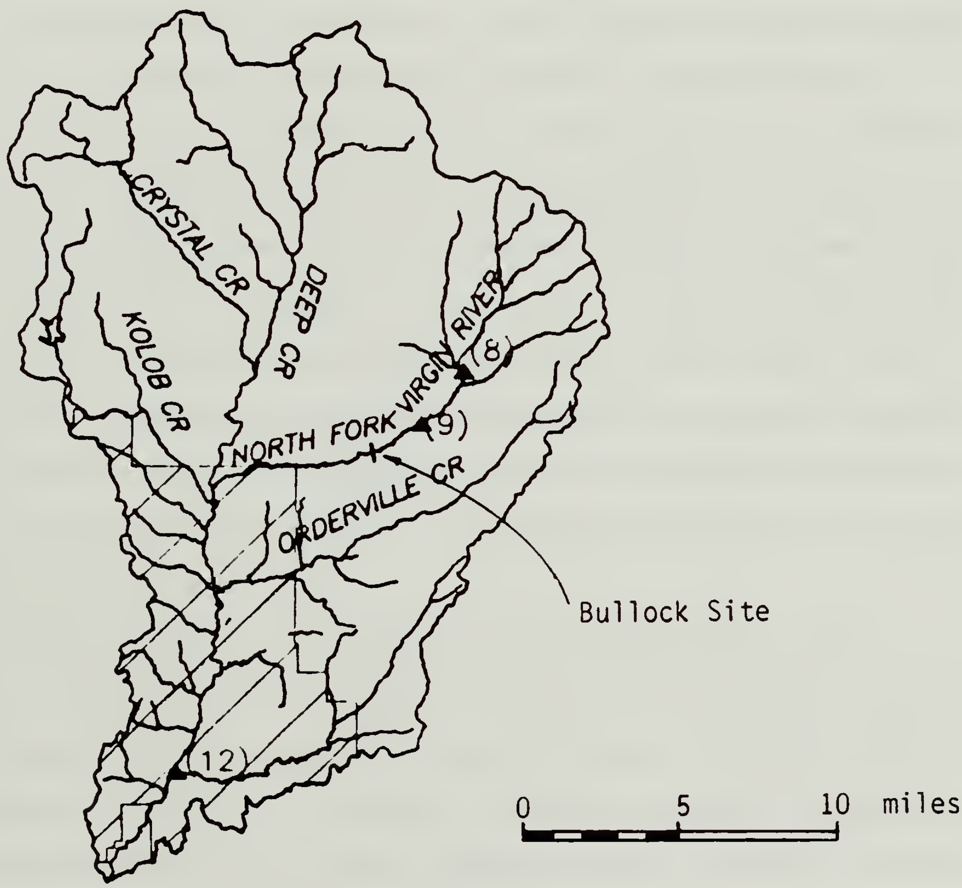


Fig.1 North Fork of the Virgin River Basin, Study Area

3.0 PRELIMINARY DATA ANALYSIS

3.1 Differences in Runoff Characteristics

The stations listed in Table 1 have observed mostly natural (virgin) streamflows, except for some diversions upstream from the gage near Springdale. There are clear differences between flow characteristics at the upper versus the lower portions of the basin. To illustrate these differences, Figure 2 displays average flow regime conditions at sites (8) and (12), for the same 10-Yrs period of record, from 1975 to 1984.

Graph 2A shows average hydrographs at both locations comprised of mean-daily flows (heavy-solid lines). Besides the marked contrast in water yield as a consequence of the difference in the size of the contributing drainage areas, the periodic standard deviation (light-solid lines) also shows marked differences. Flow variability is also shown in Graph 2B but expressed in the form of the coefficient of variation (Cv), computed as the ratio of the standard deviation of daily flows divided by their mean value. Notice the more uniform (less variance) flow pattern at site (8) in comparison to that at site (12). The more uniform flow pattern at high elevations is attributed to the predominant relatively uniform types of water sources contributing runoff, such as snowmelt producing runoff and groundwater contribution to surface flow. In contrast, the site near Springdale located at low elevation in the NFVR basin, displays a flow pattern where besides the two water sources mentioned above, a third source, rainfall, plays a very important role.

The surface runoff pattern is a result of the precipitation regime in the region. Precipitation frequency distribution at Zion national Park (Station # 9717) displays a bimodal shape, with the two peaks during the months of February and August, and with the lowest frequency during the month of June. The first peak corresponds to the winter snowfall and the second one to summer thunderstorms. Sudden produce sharp flow increases of relatively short duration. They can take place all year around, but mostly outside the snowmelt season. Thunderstorms are particularly evident during the late spring and summer seasons at the lower elevations.

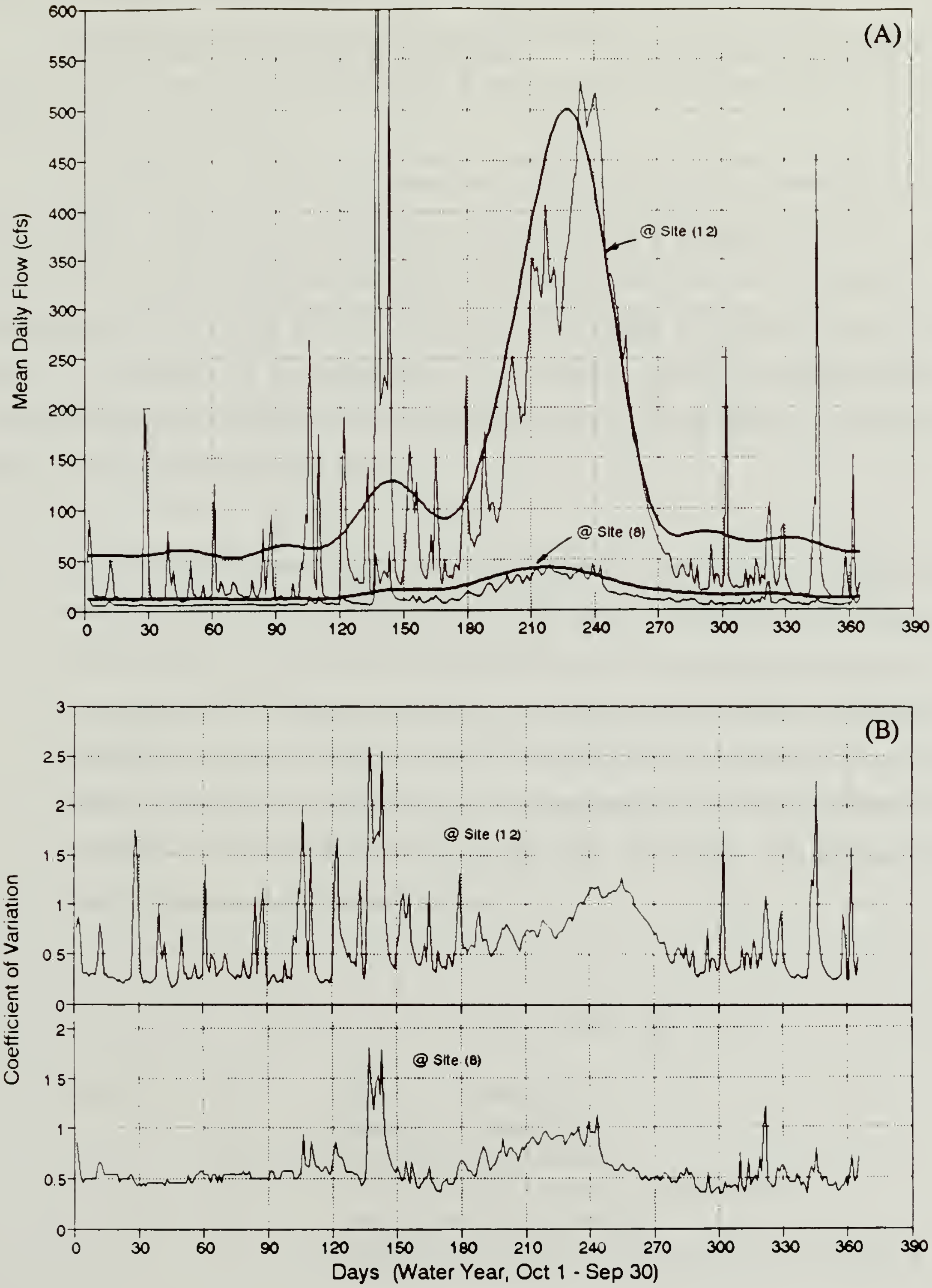


Fig.2 Runoff Characteristics at High and Low Areas of the NFVR Basin, [1975-1984] - (A) Periodic Mean and Standard Deviation, (B) Periodic Coefficient of Variation

3.2 Extension of Flow Record

To improve the reliability of the periodic mean and variance of daily flows estimated at the study site (9), its historical period of record is extended based on data at neighboring stations, sites (8) and (12).

As indicated in Table 1, 10 water-years of daily flows were collected at site (8), from 1975 to 1984. The station downstream, site (9), has only 6 years of daily flows available, from 1979 to 1984. The close proximity of the two stations and the six years of simultaneous daily records give us the opportunity to extend the record of flows at site (9), from six to ten years, in a reliable manner. The shortest record is extended based on the values registered at the station with the longest sequence during the non-concurrent period of record. The procedure is as follows:

1. First, daily flows were aggregated and regressed seasonally, i.e., corresponding to 12 monthly seasons. For instance, the 186 (31 days x 6 years) daily flow values corresponding to the month of October at site (9) are regressed versus the corresponding 186 values at site (8). Flow values were initially transformed to the logarithmic domain in order to more closely resemble a Gaussian process. Second order polynomials were selected as the most suitable relationship between the two variables, outperforming the linear model in all 12 seasons. The adopted second order regression model takes the form,

$$Y = a + bX + cX^2 + \sqrt{(1 - r^2)}\,\sigma_y\,\epsilon \qquad \text{Eq.(1)}$$

where:

Y	: dependent variable
X	: independent variable
a,b,c	: parameters of the model
r	: cross-correlation between series X and Y
σ_y	: standard deviation of series Y
ϵ	: random normal variable with mean '0' and variance '1'

Table 2.1 shows the basic statistics of daily flows (after the Log-transformation and monthly aggregation)and the cross correlation value of the corresponding series, which is a dimensionless measure of the correlative association between the two series. The right portion of Table 2.1 contains the parameters of the regression model and the coefficient of determination r^2, which indicates the proportion of the variance of Y explained by the regression function. The twelve sets of regressed values and their corresponding fitted models were plotted as shown in Appendix I. Notice that in order to explain all the variance of the dependent variable, a random component has been added to the regression term. In this manner, the extra variance ($1-r^2$), not explained by the pure regression term, is introduced in the right-hand side of Eq.(1).

2. Flows at sites (8) and (9) were also correlated at the monthly and yearly levels, which, as expected, show an even higher degree of association than at the daily level. Results of the linear models adopted at the monthly and yearly levels are shown in Tables 2.2 and 2.3 respectively. Estimated monthly and yearly flows at site (9) during the period 1975-1979, based on flows measured at site (8) during that same period, were used to adjust the regressed daily values at site (9), such that monthly volumes first, and annual volumes second, were also preserved.

In summary, as a result of the record extension procedure described above, a total of ten water-years of mean-daily flows became available at site (9), NFVR above Narrows Canyon, from 1975 to 1984.

Table 2.1 Estimation of Basic Flow Statistics and Results of Second Order Regression Analysis of Daily Flows (in cfs) at Site (9) versus Site (8), Period [1979-1984].

Month	[Y]=Site (9) Mean	St.Dv	Skew	Kurt.	[X]=Site (8) Mean	St.Dv	Skew	Kurt	Cross Correl.	Polynomial Coefficients (a)	(b)	(c)	r^2
Oct	2.740	0.320	-0.200	3.504	2.791	0.296	0.382	3.291	0.754	1.5586	0.0334	0.1382	0.58
Nov	2.776	0.304	-0.055	2.377	2.767	0.310	-0.406	1.925	0.855	4.5500	-2.2362	0.5694	0.77
Dec	2.743	0.381	-0.401	2.561	2.675	0.387	0.217	1.872	0.909	-0.5829	1.5914	-0.1275	0.84
Jan	2.647	0.540	0.224	2.070	2.679	0.499	0.355	2.175	0.938	0.5749	0.5270	0.0889	0.89
Feb	2.922	0.636	1.460	6.042	2.929	0.632	1.310	5.563	0.969	0.5788	0.6456	0.0503	0.95
Mar	3.293	0.344	0.186	2.744	3.227	0.312	0.339	3.236	0.942	-0.5024	1.3057	-0.0398	0.90
Apr	3.714	0.599	0.397	1.788	3.637	0.587	0.374	1.782	0.957	0.7361	0.6557	0.0437	0.93
May	3.701	0.838	-0.003	1.610	3.704	0.785	-0.023	1.757	0.975	-0.1006	1.0037	0.0058	0.96
Jun	3.215	0.616	0.377	2.293	3.284	0.446	0.520	2.437	0.967	-1.7090	1.6519	-0.0457	0.95
Jul	2.851	0.474	0.155	2.377	2.913	0.335	0.483	3.084	0.893	-1.3888	1.6345	-0.0605	0.81
Aug	2.796	0.564	-0.341	4.726	2.872	0.407	0.698	5.308	0.889	-2.1913	2.2085	-0.1611	0.81
Sep	2.506	0.476	0.196	2.297	2.642	0.381	0.176	2.757	0.879	-0.3013	1.0181	0.0164	0.78

Note: Daily-flow values were aggregated by month and transformed to the Log-domain

Table 2.2 Results of Linear Regression Analysis of Monthly Flows (in cfs) at Site (9) versus Site (8), Period [1979-1984].

Month	(a)	(b)	r^2
Oct	1.28717	0.89086	0.729
Nov	1.52828	0.89931	0.847
Dec	2.47102	0.96898	0.912
Jan	-0.61087	1.02078	0.973
Feb	-1.18490	1.04869	0.991
Mar	-5.52688	1.29032	0.939
Apr	-2.12857	1.13887	0.987
May	-1.81971	1.06110	0.981
Jun	-10.4163	1.37920	0.988
Jul	-8.71912	1.44201	0.928
Aug	-6.67712	1.33900	0.936
Sep	-2.48268	1.09339	0.848

Table 2.3 Results of Linear Regression Analysis of Annual Flows (in cfs) at Site (9) versus Site (8), Period [1979-1984].

Year	(a)	(b)	r^2
	-4.1736	1.19213	0.965

4.0 LONG-TERM FLOW CHARACTERISTICS AT THE STUDY SITE

4.1 Estimation of Mean Annual Discharge

In general, it is accepted that given a period of flow record extending for at least 10 years, the analyst can compute reasonably good estimates of the long-term flow characteristics at the site. Unfortunately, this should not be considered the case at our study site. Reviewing Figure 6 from the previous report, which shows annual discharge for the NFVR near Springdale for the period of record [1926-1991], it can be inferred that the 10-yrs subperiod from 1975 to 1984, as an average, it would indicate a water availability considerably larger than what it should be expected in the long run. The same information is repeated below in Figure 3, where the subperiod [1975-1984] has been separated from the remaining record for visualization purposes.

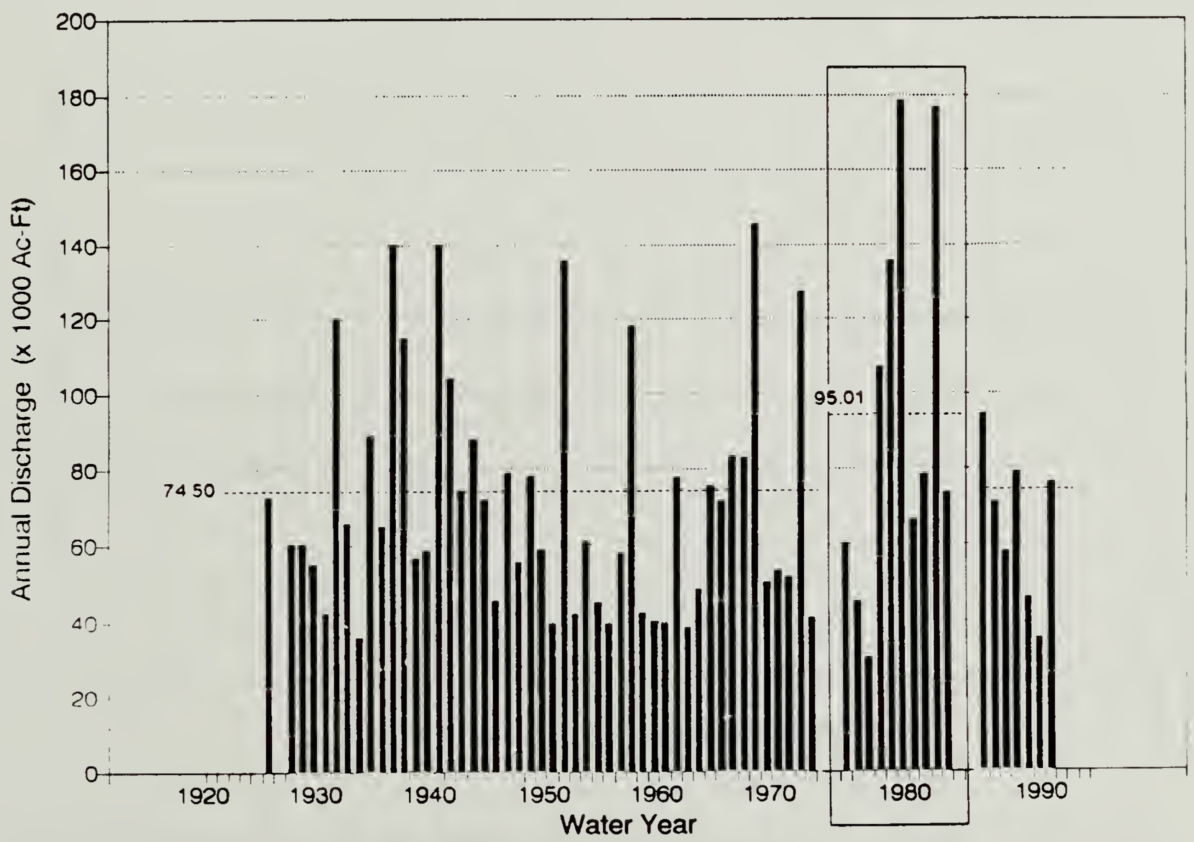


Fig.3 Annual Discharge at the NFVR Near Springdale, Site (12)

Notice that during those ten years, the two wettest (1980 and 1983) and the driest (1977) water-years on record took place. The average annual discharge for the 10-Yrs, 95,010 Ac-Ft, is a 27.5% larger than the 65-Yrs average, 74,500 Ac-Ft, at site (12). Then it can be hypothesized that similar flow conditions existed in the upper portion of the basin, and consequently, that the mean annual discharge computed from the short record at station 09405450 is not representative of the long-term flow conditions at site (9).

To confirm the hypothesis above, annual discharge for the NFVR at sites (9) and (12) were correlated for the 10-Yrs period with concurrent data (recall earlier extension of flow record at site (9) from 6 to 10 Yrs.). The 10 pairs of data and the adopted linear regression function are shown in Figure 4. As expected, annual flows measured near Springdale and

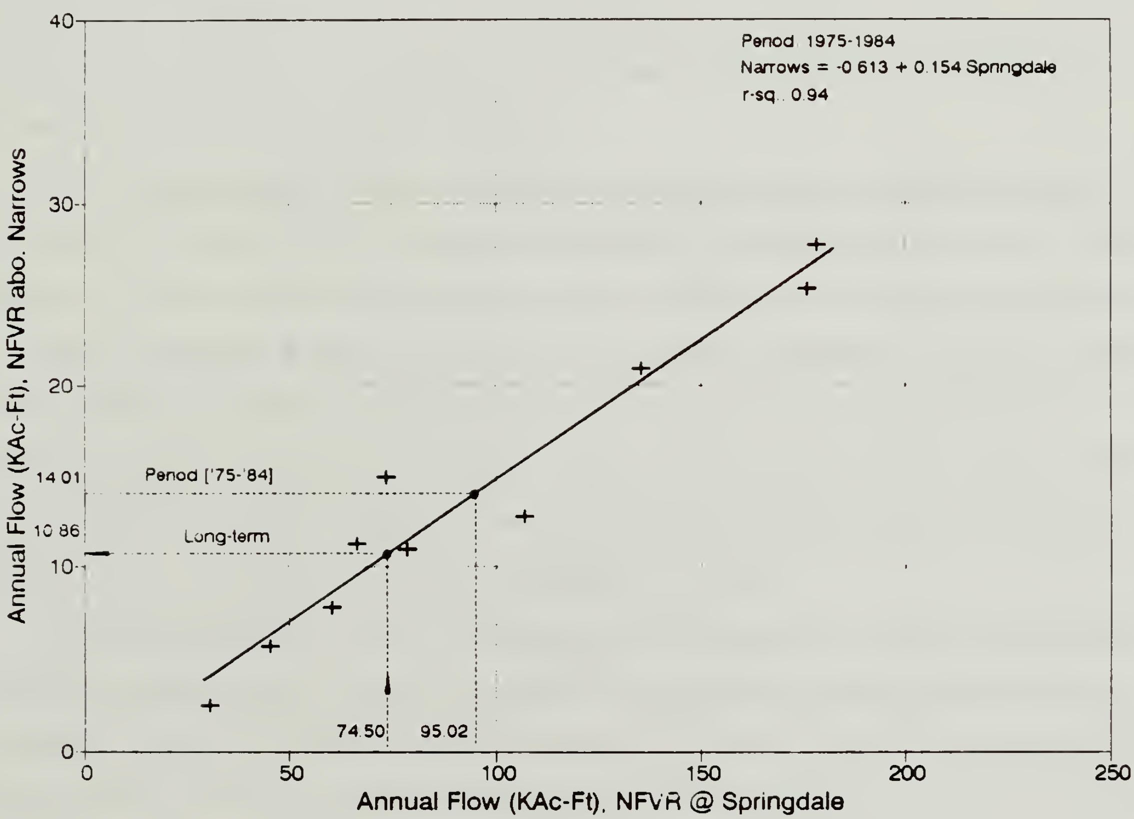


Fig.4 Regression of Annual Flows at Site (9) versus Site (12), Period [1975-1984].

above Narrows Canyon show a high linear correlation, yielding a coefficient of determination r^2 of 0.95. This high value of r^2 is even more meaningful when considering that the correlated values cover the whole range of annual flows ever registered at Springdale, from the driest to the wettest water-years. The linear relationship in Figure 4 can therefore be utilized to estimate the long-term mean annual discharge for the NFVR above Narrows Canyon, computed as 10,860 Ac-Ft (15.03 cfs), based on the measured 65-Yrs mean annual discharge near Springdale of 74,500 Ac-Ft.

4.2 Estimation of Periodic Mean and Periodic Standard Deviation

Once the long-term mean annual discharge at site (9) was estimated, the following step was to estimate the long-term periodic mean and periodic standard deviation of daily flows. The procedure adopted for this task was similar to the one adopted to transfer information from site (8) to site (9). But this time, the correlation of daily flows was carried out between sites (9) and (12). Site (12) is the closest location to site (8) with a relatively long period of record of daily flows, 62 years.

In the same way done earlier in Section 3.2, after estimating a complete month of daily values (by means of the daily regression equations), daily flows were adjusted (when necessary) to match the estimated flow value at the monthly level. Similarly, once a whole year of daily flows were generated, they were also subject to adjustment in order to conform with the estimated discharge at the annual level. As a result of this sequential flow adjustment procedure, the extended database of daily flows at site (9), from 1926 to 1988 (1927 excluded), yields an average mean annual discharge practically equal to the value found in Figure 4 , 10,860. Ac-Ft (or its equivalent 15.26 cfs).

To help visualize the change in average annual discharge as a result of the length of the period of analysis being considered, Figure 5 displays three different average annual hydrographs. Each one of them represents average daily flows based on the three subperiods discussed so far, [1979-1984], [1975-1984] and [1927-1988], the last one also named "long-term". The hydrographs shown in Figure 5 are the smooth representation of the actual periodic functions, after being fitted by Fourier series.

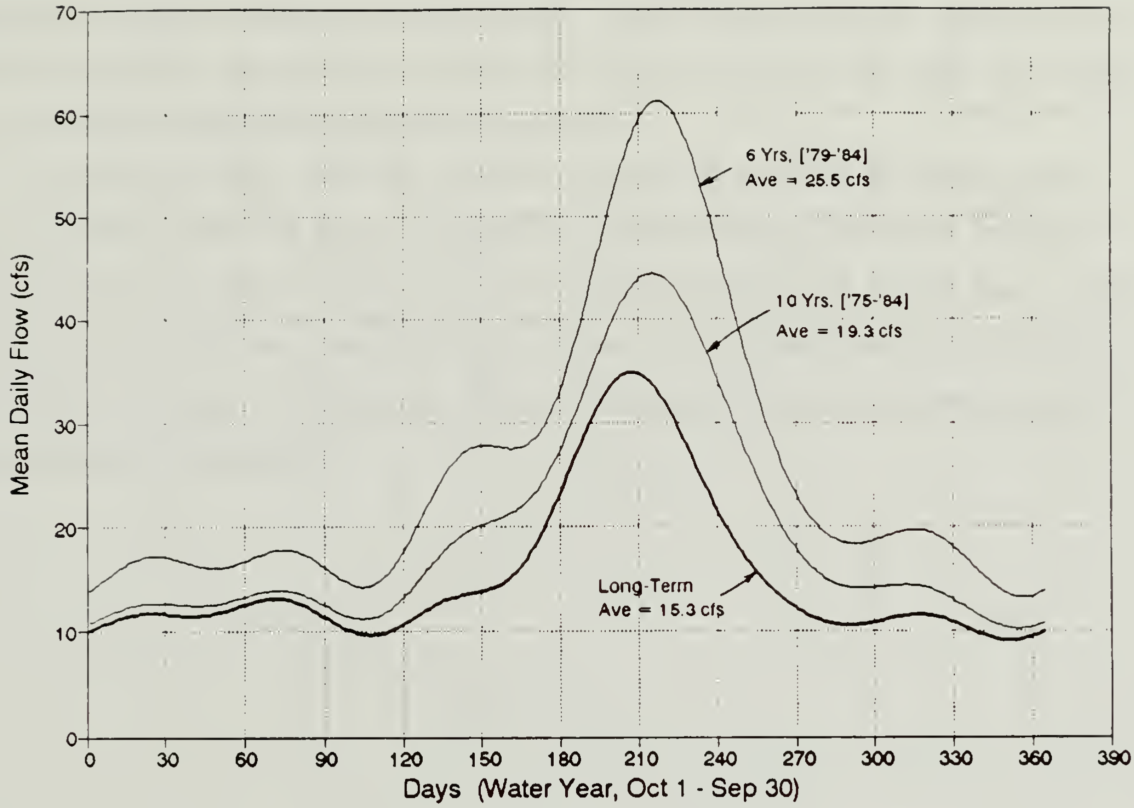


Fig.5 Average Annual Hydrographs at Site (9) as a Function of the Period of Analysis

Notice that as the length of the period of analysis increases, the mean annual discharge, i.e., the area under the curve, decreases. This is consistent with the findings from Figure 3, which shows extremely wet water-years during the 6-Yrs subperiod, still wet conditions for the 10-Yrs subperiod, and the approximation to long-term (normal) conditions when the period of 62-Yrs is being considered. Moreover, the hydrographs become smother when going from the shorter to the longer period of record as a result of less sampling variability.

Another important aspect when analyzing the structure of daily runoff is the variability of daily flows around its average value. To compare the degree of flow variability between measured and regressed values, the periodic coefficient of variation (Cv)

was computed from both series. There were not substantial differences between the Cv values computed from the 10-Yrs and 62-Yrs subperiods, although the longer series shows slightly higher values during the summer season. These higher values can be attributed to the effect of summer thunderstorms registered at site (12), that to some extent, are carried along by the flow regression procedure into site (9).

In summary, either one of the periodic Cv functions provides reasonably good estimates of the variance of flows, and therefore, can be used to estimate the long-term standard deviation at Site (9). Figure 6 shows the adopted long-term sample mean, sample standard deviation and their corresponding smoothed periodic functions for the NFVR site above Narrows Canyon. The numeric values of the periodic functions shown in Figure 6 were included as Appendix II.

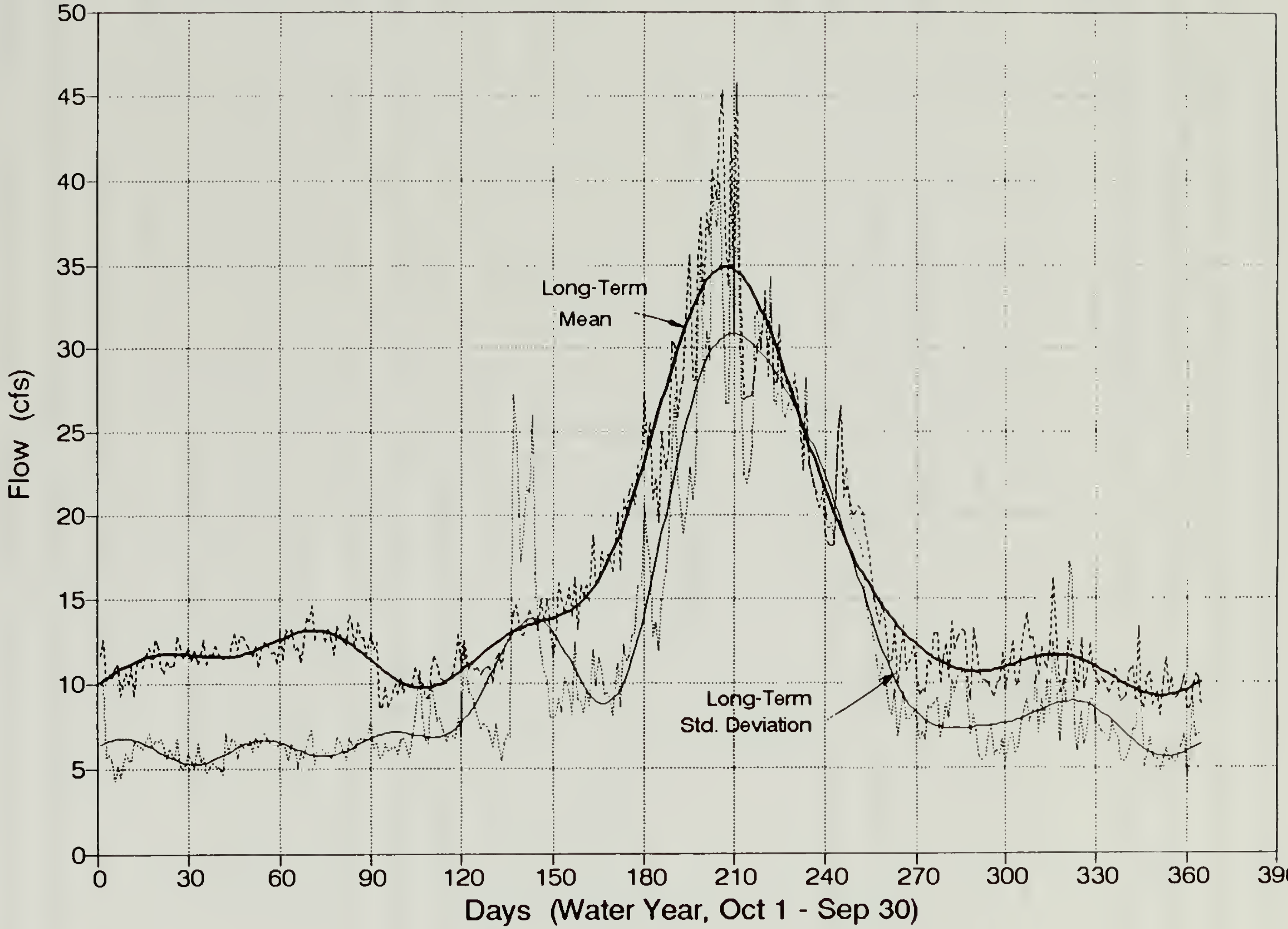


Fig.6 Long-Term Periodic Mean and Standard Deviation of Daily Flows for the NFVR Above Narrows Canyon

5.0 DEPENDENCE STRUCTURE OF DAILY FLOWS

The understanding of the dependence structure of short interval time series is crucial for the stochastic modeling of daily time series. The periodicity in parameters that describe or measure the dependence of successive values of daily flows, k-lag apart, can be studied from a physical and a statistical point of view. The latter is the main subject of investigation in this section.

The time dependence structure of daily runoff is a highly site-specific flow characteristic. For that reason, the 10-Yrs period of record [1975-1984] at Site (9) was adopted to study the dependence structure of daily-flows in the upper region of the NFVR basin. The 10-Yrs period, although short, it should provide a good representation of flow sequencing due to the alternation of very dry, very wet and normal water-years during that particular period.

5.1 Normalization and Standardization of Daily Flows

In studying the properties of autocorrelative dependence of flows decisions should be made on whether to study the structure of the original flow series, or alternatively, "standardized" and/or "normalized" transformations of the flow series. For instance, when analyzing the periodic skewness and kurtosis coefficients of daily flows at site (12), see Figures 13 and 14 in the previous report, it becomes apparent that the marginal distributions of daily flows change from one day to another, yielding a (periodic) highly non-stationary process. Moreover, the values of the skewness and kurtosis coefficients depart substantially from zero and three respectively, indicating how different these distributions are from the normal distribution. This typical departure from normality of daily flows is also observed at site (9), making the structural models available, developed for Gaussian processes, unapplicable for the case of daily flows.

A way of circumventing the problem of lack of normality of daily flows is to find an appropriate transformation of the asymmetric series, such that the transformed series is closer to normality. Typically in hydrology, the logarithmic, the power and the Wilson-Hilferty functions are used to normalize daily flows. The normalized series $Y_{\upsilon,\tau}$ is obtained as

$$Y_{\upsilon,\tau} = f_{\tau}(X_{\upsilon,\tau}) \qquad \text{Eq.(2)}$$

where the function f_τ is chosen based on the skewness test of normality.

On the other hand, standardization of a series removes the within the year seasonal pattern of its means and standard deviations. The standardized series is computed by

$$Z_{\upsilon,\tau} = \frac{(X_{\upsilon,\tau} - m_\tau)}{s_\tau} \qquad \text{Eq.(3)}$$

for $\upsilon=1,2,...N$ with N the number of years, and $\tau=1,2,...,w$ with $w=365$ the number of time intervals per year. Equation(3) yields the standardized variable $Z_{\upsilon,\tau}$ where m_τ and s_τ are the periodic means and standard deviation respectively of the series being standardized. This transformation restores stationarity in the two parameters, yielding a transformed process with approximately mean zero and variance one. Third and fourth order moments remain unaffected after the standardization. Depending on the characteristics of the flow series at hand, standardization is applied before or after normalization. Table 3 presents results from all the attempted transformations of daily flows at site (9) during the period [1975-1984]. Results are presented as average for the 365 values of the periodic mean, periodic standard deviation, periodic skewness and periodic kurtosis coefficients of the resulting series after the transformation . First case implies no transformation (original domain).

Table 3 Average Statistics After Transforming Daily Flows Series at Site (9)

Type of Transformation	Average value of periodic ... m_τ	s_τ	g_τ	k_τ	# Days Rejected 99% C.L.
$Y_{\upsilon,\tau} = X_{\upsilon,\tau}$	19.34	14.92	0.94	4.61	148
$Y_{\upsilon,\tau} = \text{Log10}(X_{\upsilon,\tau})$	1.12	0.36	-0.55	4.76	128
$Y_{\upsilon,\tau} = \text{Log\_e}(X_{\upsilon,\tau})$	2.58	0.83	-0.55	4.76	128
$Y_{\upsilon,\tau} = \text{Sqrt}(X_{\upsilon,\tau})$	4.00	1.54	0.30	4.03	37
$Y_{\upsilon,\tau} = \text{Log Wilson-Hilferty}(X_{\upsilon,\tau})$	0.00	1.01	0.01	4.11	35

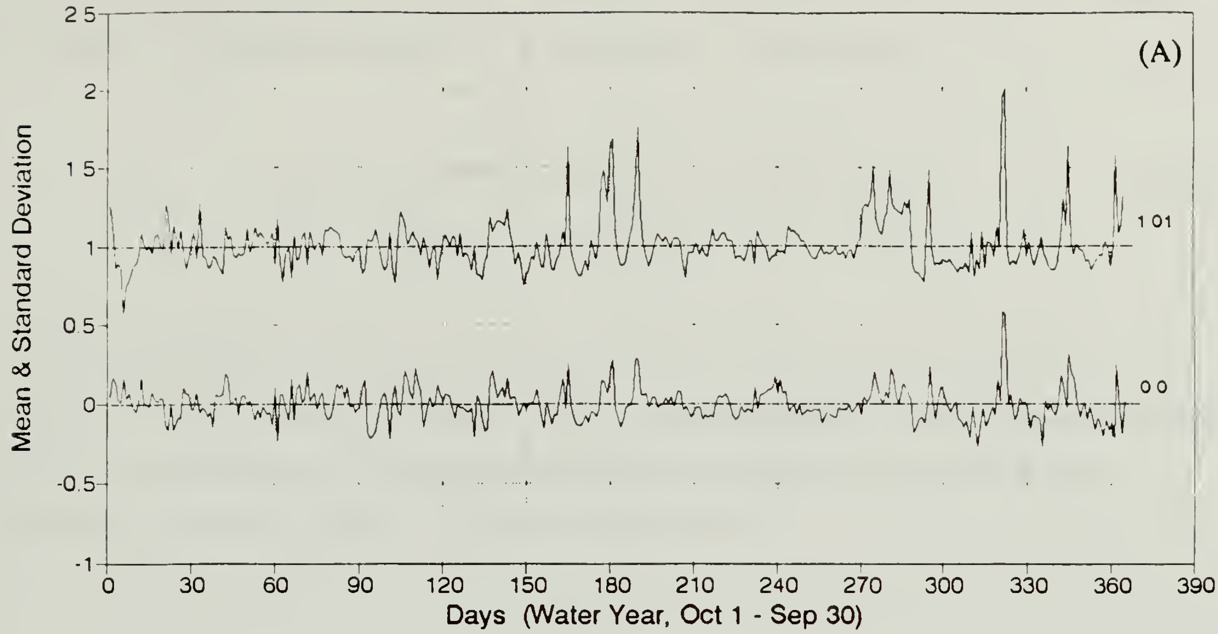
Table 3 indicates the Log Wilson-Hilferty (LWH) as the most suitable transformation, yielding the smallest number of days rejecting the normality hypothesis, with 35days outside the 99% confidence limits. Unlike the other transformations where the series is first normalized and then standardized, LWH involves first transforming the original series into the logarithmic domain as in Eq.(2). Then, the series is standardized using Eq.(3), where m_τ and s_τ are computed in the Log-domain. Finally, the Wilson-Hilferty transformation is carried out using Eqs.(4) and (5) to arrive at $Z_{\upsilon,\tau}$. Matalas (1967) gives

$$Z_{\upsilon,\tau} = \frac{6}{G_\tau(y)}\left[\left(\frac{G_\tau(y)\, Y^*_{\upsilon,\tau}}{6} + 1\right)^{1/3} - 1\right] + \frac{G_\tau(y)}{6} \qquad \text{Eq(4)}$$

valid for values of the skewness $G_\tau(y) \neq 0$. If $G_\tau(y) = 0$ no transformation is necessary. The Fourier fitted series of the daily-skewness coefficients in the log(e) domain is used for the LWH transformation. Furthermore, $Y^*_{\upsilon,\tau}$ is given by McGinnis and Sammons (1970)

$$Y^*_{\upsilon,\tau} = \begin{cases} \max[Y_{\upsilon,\tau}, -2/G_\tau(y)] & \ldots \text{ if } G_\tau(y) > 0 \\ \min[Y_{\upsilon,\tau}, -2/G_\tau(y)] & \ldots \text{ if } G_\tau(y) < 0 \end{cases} \qquad \text{Eq.(5)}$$

After completing the LWH transformation, normality and stationarity of the standardized series is reached for all practical purposes. Figure 7(A) shows the periodic first two moments of the LWH transformed series. Notice that the periodic patterns were totally removed from both series, with mean zero (average) and variance one (average) respectively, as a result of the standardization . Also, the LWH transformation produced average coefficients of skewness, Figure 7(B), and kurtosis which are close to zero and three respectively, as expected for the normal marginal distributions, regardless of their small variations around the averages. Due to the relatively small number of years of record available for the analysis and the large number of time intervals under consideration, the Fourier series fitting of the periodic characteristics of the historical series is desirable. The parametric method of normalization allow us to decrease the number of parameters of the periodic components and to smooth out sampling variability.



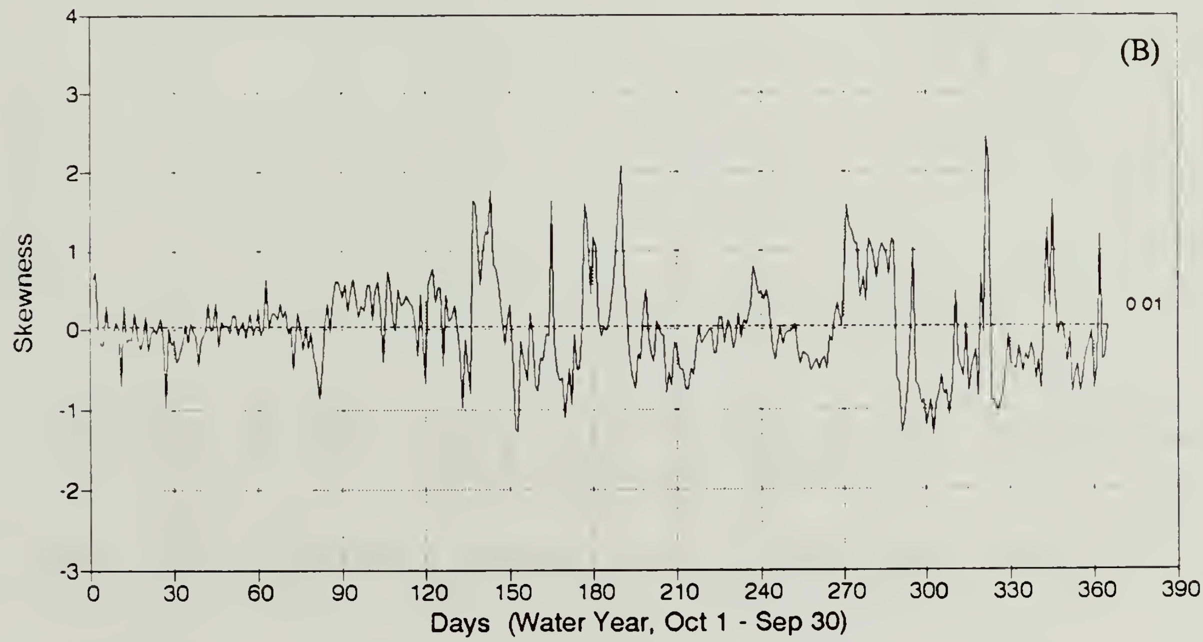


Fig.7 Periodic Statistics of Daily-Flows at Site (9) in the Log Wilson-Hilferty Domain.
(A) Mean and Standard Deviation, (B) Skewness Coefficients

5.2 Analysis of Dependence Structure

The seasonal serial correlation coefficients $r_{k,\tau}$ of daily flows at site (9) is displayed by Figure 8. The periodic function $r_{k,\tau}$ is estimated by the expression

$$r_{k,\tau} = \frac{\sum_{\upsilon=1}^{N} (X_{\upsilon,\tau} - m_\tau)(X_{\upsilon,\tau+k} - m_{\tau+k})}{\left[\sum_{\upsilon=1}^{N} (X_{\upsilon,\tau} - m_\tau)^2 \sum_{\upsilon=1}^{N} (X_{\upsilon,\tau+k} - m_{\tau+k})^2\right]^{1/2}} \qquad \text{Eq.(6)}$$

where all variables have been previously defined. The most important case is for k=1, the periodic first autocorrelation coefficient. The $r_{1,\tau}$ function definitely shows much less within the year variation than the equivalent periodic function computed for the NFVR near Springdale, as shown in Figure 15 of the previous report.

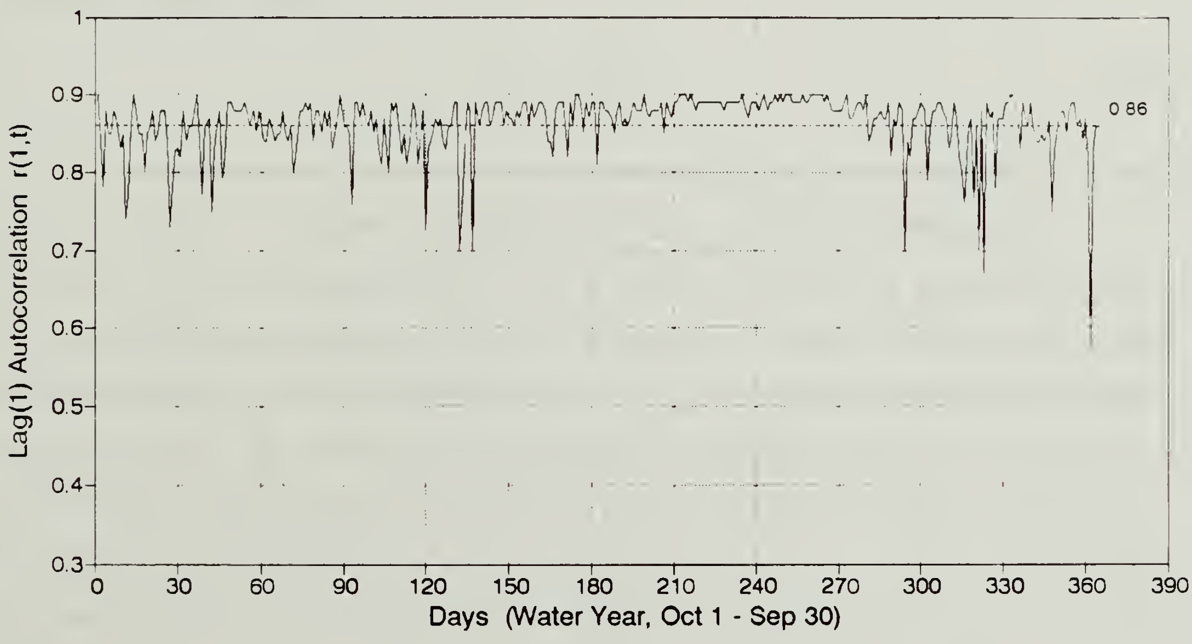


Fig.8 Lag-1 Autocorrelation Function of Standardized Daily Flows at Site (9)

The small variance shown by $r_{1,\tau}$ is displayed mostly outside from the part of the year in which the snowmelt is the major runoff contributor (τ=180-280). Notice that the majority of the values are contained within a relatively narrow range (0.8-0.9).

Furthermore, the periodicity in $r_{1,\tau}$ can be considered negligible in practical terms. In other words, it can be safely assumed that the lag-1 serial autocorrelation of daily flows at site (9) do not significantly vary from day-to-day, and therefore, it can be represented by a uniform value of $r_{1,\tau} \approx 0.86$ for $\tau = 1, 2, \ldots, 365$. Besides $r_{1,\tau}$, autocorrelations functions for other lags were also computed (though not shown in Figure 8). $r_{k,\tau}$ functions for k=1,2 and 3 resulted almost undistinguishable among themselves. This finding give us an early indication of the uniformity and persistence of mean-daily flows at high elevation watersheds of the NFVR hydrologic region.

Because of the negligible periodicity in the serial autocorrelation of flows, the seasonal time series $Z_{\upsilon,\tau}$ can be lumped into a non-periodic series Z_τ, with τ=1,2 ... N', where N'=N . w. For the case under consideration, N'=10 Yrs x 365 days/Yrs=3650 occurrences of the Z variable. Consequently, the k^{th} autocorrelation coefficient is computed as a function of lag (k) only

$$r_k = f(k) \qquad \text{Eq.(7)}$$

The computation of r_k can still be carried out by Eq.(6), used to compute $r_{k,\tau}$, except that the time index τ is dropped. Figure 9 shows the r_k function, known as the correlogram, for the standardized series of daily flows at site (9) in the original flow-domain, which uses the mean and standard deviation of the entire $Z_{\upsilon,\tau}$ series. Note the very smooth and monotonic decay of the correlogram from r_1 to r_{390}, with values greater than zero even at the end of the year. The slow decay rate indicates a hydrologic system with very long water storage memory. The positive time dependence of r_k also explains the large persistence of flows of the same type, that is, low values mostly followed by low values, and conversely, high values followed by high values. The correlogram shows no indication of any remaining periodicity in the standardized series. The r_1 value of the correlogram is 0.956. The same r_1 value computed in the Log Wilson-Hilferty domain drops slightly to 0.938.

To contrast the difference in flow dependence structure between the upper and the lower regions of the NFVR basin, the correlogram of daily flows at site (12), during the same 10-Yrs period, was also included in Figure 9. Notice the much faster decay of r_k at site (12), particularly during the first 10 lag-periods. The larger influence of precipitation in

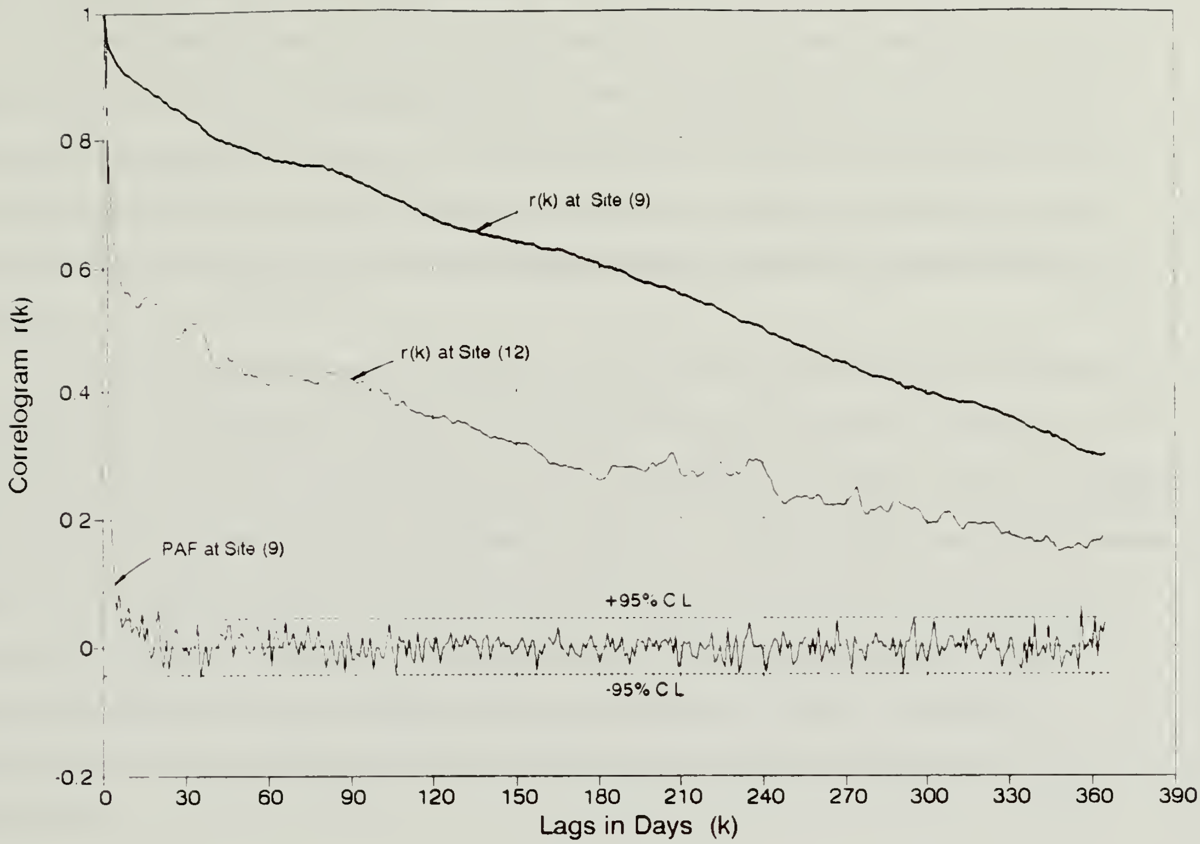


Fig.9 Correlogram of Standardized Daily Runoff at Sites (9) and (12) - Flow Domain

the form of rainfall at the lower elevations is the main factor responsible for the difference in the correlation structure of the residual series between the two areas.

Another tool for analyzing flow dependence structure is the partial autocorrelation function (PAF), also displayed in Figure 9, for site (9). PAF measures the degree of linear association between successive autocorrelation coefficients. Together with the correlogram, help the analyst in the identification of the order of the linear dependence model that more closely will resemble the flow structure of the residual series. The first eight PAF coefficients are significant at the 95 percent probability levels, given approximately by $\pm 2/\sqrt{N'} = 0.033$. Then, it oscillates within the confidence band. These results would suggest a stochastic model with a relatively large number of autorregresive terms.

5.3 Identification of the Flow Generation Model

As indicated in Section 5.1, the periodicity in the mean and standard deviation of daily flows was removed by using a Fourier series representation of Y_τ and s_τ and by creating the standardized series $Z_{\upsilon,\tau}$. By removing the periodicity in the mean and the standard deviation, the series $Z_{\upsilon,\tau}$ became second-order stationary, provided the serial autocorrelation function can be considered approximately stationary as demonstrated in Figure 8.

The typical flow pattern of the NFVR, with low flows mainly from groundwater contribution and higher flows produced by rainfall events and snowmelt, demands a flow generation model that it can accommodate this mixed behavior. Models with autoregressive (AR) and moving average (MA) components, known as ARMA models, are recommended under this conditions. More specifically, ARMA models with constant parameters are expected to perform satisfactory modeling mean-daily flows, provided that the flow correlation function is approximately stationary (Salas et al., 1985). The mixed autoregressive-moving average model for fitting the periodic hydrologic series $Y_{\upsilon,\tau}$ is expressed by

$$Y_{\upsilon,\tau} = m_\tau + s_\tau Z_{\upsilon,\tau} \qquad \text{Eq.(8)}$$

where $Z_{\upsilon,\tau}$ will be represented by an ARMA(p,q) model with (p) autoregressive parameters and (q) moving average parameters, constant in time,

$$Z_\tau = \sum_{j=1}^{p} \phi_j Z_{\tau-j} - \sum_{i=1}^{q} \theta_i \epsilon_{\tau-i} + \epsilon_\tau \qquad \text{Eq.(9)}$$

where ϕ_j, $j=1,2,...p$ and θ_i, $i=1,2,...q$ are the coefficients of the model to be evaluated from data, and ϵ_τ is an independent normal variable. The visual display of the original series and the behavior of the autocorrelation function coupled with that of the PAC have revealed that a relatively large number of ϕ's and θ's parameters will be required.

The estimation of the ARMA(p,q) parameters is a complex process. Details of the estimation procedure are not provided here, however they can be found in Salas et al.(1985,

pp.223). Preliminary estimates of the parameters of tentative models were obtained by the Method of Moments, and a posteriori, more accurate estimates of the parameters were obtained by the Non-linear Least-square algorithm. Computations were performed by a computer program. Analysis of a parameter estimation output is analyzed in Section 5.4.

Several competing ARMA(p,q) models were fitted to the standardized Z_τ series. The Akaike Information Criterion (AIC), Akaike (1974), a test used to select the best among competing models was utilized to select the orders (p) and (q). The AIC is computed by

$$AIC = N\, Ln(\sigma_\epsilon^2) + 2\,(p+q) \qquad \text{Eq.(10)}$$

where N is the sample size and σ_ϵ^2 is the variance of the estimated residuals series obtained after fitting the prospect ARMA(p,q) model to the data series. The model that has the minimum AIC value is the theoretical choice. The model identification procedure started with low order models, for instance an ARMA(1,1), and it continued progressively overfitting the time series until the minimum AIC value was obtained. At the same time, the goodness-of-fit of the prospect model was verified. The analysis showed values of AIC (and σ_ϵ^2) relatively similar for models with AR terms greater or equal than three. However, only an ARMA(4,2) satisfied the the Parsimony Conditions. Consequently, an ARMA(4,2) model was adopted for further analysis.

5.4 Diagnostic Checking

After estimating the parameters of the identified model, it is necessary to verify the validity of the underlying assumptions. The following program output provides the values of the parameters ϕ_j and θ_i, checks the parsimony of the model parameters and tests the residuals series for independence and normality. All these requirements are derived from the estimation procedures that are commonly used in structural analysis of hydrologic processes. The details of these tests may be found in Salas et al.(1985). All conditions were accepted except the hypothesis of normality of the residuals ϵ_τ, rejected by the skewness test of normality. Daily streamflows typically display this departure from Gaussian processes. However, in this particular case, the results of the normality test can be ignored since the normality requirement of the transformed series was already verified (after LWH, Table 3).

Table 4. Parameter Estimation and Diagnostic Check for ARMA(4,2) Model

Project	:	Zion National Park
Name of River Basin	:	North Fork of Virgin River
USGS gaging Station	:	09405450 - NFVR above Narrows Canyon
Type of Data	:	Mean Daily Flows
Data Units	:	No Units, Variable Standardized and Normalized using Log_Wilson-Hilferty Transformation
Period of Analysis	:	1975-1984

Statistical Characteristics of the Dependent Variable

> Type of series : Standardized/Normalized(LWH)/Stationary
> Mean of the Series : -0.001
> Standard Deviation : 0.979
> Skewness Coefficient : 0.252

> Sample Autocorrelation Function

k	-0.95CL	r(k)	+0.95CL
1	-0.032	0.938	0.032
2	-0.032	0.912	0.032
3	-0.032	0.899	0.032
4	-0.032	0.884	0.032
5	-0.032	0.873	0.032
6	-0.032	0.867	0.032
7	-0.032	0.862	0.032

k	-0.95CL	r(k)	+0.95CL
8	-0.032	0.856	0.032
9	-0.032	0.849	0.032
10	-0.032	0.840	0.032
50	-0.032	0.734	0.032
100	-0.032	0.659	0.032
200	-0.032	0.536	0.032
300	-0.032	0.346	0.032

Parameter Estimation of AR and MA Parameters

> Moment Estimators

Par.No.	AR	MA
1	1.000	0.387
2	0.000	0.128
3	0.000	
4	0.000	

..Residual Variance:0.103

> Least Squares Estimators

Par.No.	AR	MA
1	1.091	0.505
2	0.157	0.350
3	-0.147	
4	-0.103	

..Residual Variance:0.099

Stationarity and Invertibility Check for ARMA Models (Parsimony Test)

> AR Parameters

Par.No.	AR Value	Root
1	1.096	2.234
2	0.188	1.979
3	-0.207	1.913
4	-0.080	1.145

..Stationarity Condition is Accepted

> MA Parameters

Par.No.	AR Value	Root
1	0.501	-1.709
2	0.361	-1.673

..Invertibility Condition is Accepted

A Portmanteau Lack of Fit Test for Independence of Residuals

```
  Computed Critical Value            :  834.9
  Chi-square Tabulated Value         :  976.9
  Degrees of Freedom                 :  906
  Confidence Level                   :  0.950
  -----------------------------------------------
  ..Hypothesis of Independence is Accepted
```

Skewness Test for Normality of Residuals

```
  Tabulated Test Value        :   0.067
  Computed Test Value         :   2.180
  Probability Level           :   0.950
  ---------------- ---------------------
  ..Hypothesis of Normality is Rejected
  ..Recommendation ==> Transform Series
```

Overfitting Check Using the Akaike Information Criterion

```
  AIC of ARMA (4,2) Model   :  -8428.7
  AIC of ARMA (5,2) Model   :  -8428.9
  AIC of ARMA (4,3) Model   :  -8405.7
  AIC of ARMA (3,2) Model   :  -8398.7
  AIC of ARMA (4,1) Model   :  -8421.6
```

6.0 GENERATION OF SYNTHETIC FLOWS

6.1 Data Generation Scheme

The flow generation process is started by using Eq.(9) recursively to generate synthetic values of Z_τ. The computational process is initiated by arbitrarily assigning (p) initial values of 'Z' and computing (q) initial values of 'ϵ' . The first generated 50 values of Z_τ are discarded to avoid the transient effect of the initial conditions. Rewriting Eq.(9) according to the identified ARMA(4,2) model, the following expression results

$$Z_\tau = \sum_{j=1}^{4} \phi_j Z_{\tau-j} - \sum_{i=1}^{2} \theta_i \epsilon_{\tau-i} + \sqrt{0.099}\, \xi_\tau \qquad \text{Eq.(11)}$$

with parameters ϕ_j , θ_i and σ_ϵ obtained from Table 4. ξ_τ is the standardized normal variable with mean zero and variance one, generated using the Box-Muller formula written as

$$\begin{aligned} e_1 &= [2\, Ln(1/u_1)]^{1/2} \cos(2\pi u_2) \\ e_2 &= [2\, Ln(1/u_1)]^{1/2} \sin(2\pi u_2) \end{aligned} \qquad \text{Eq.(12)}$$

where u_1 and u_2 are two independent uniformly distributed (0,1) random numbers. A subprogram that generates normal random numbers was made part of the flow generation model.

Having obtained the sequence of $Z_{\upsilon,\tau}$, the backward Wilson-Hilferty transformation is applied using

$$Y^*_{\upsilon,\tau} = \frac{2}{G_\tau(y)} \left\{ \frac{G_\tau(y)}{6} \left[Z_{\upsilon,\tau} - \frac{G_\tau(y)}{6} \right] + 1 \right\}^3 - \frac{2}{G_\tau(y)} \qquad \text{Eq.(13)}$$

in which $G_\tau(y) \neq 0$, where $G_\tau(y)$ is the skewness coefficient of the daily series in the Log(e) domain. The variable $Y^*_{\upsilon,\tau}$ is further transformed by

$$
Y_{\upsilon,\tau} = \begin{cases} \max[Y^{*}_{\upsilon,\tau}, -2/G_{\tau}(y)] & \ldots \ if\ G_{\tau}(y) > 0 \\ \min[Y^{*}_{\upsilon,\tau}, -2/G_{\tau}(y)] & \ldots \ if\ G_{\tau}(y) < 0 \\ Y_{\upsilon,\tau} = Z_{\upsilon,\tau} & \ldots \ if\ G_{\tau}(y) = 0 \end{cases} \qquad \text{Eq.(14)}
$$

$Y_{\upsilon,\tau}$ becomes the generated data in the Log-domain (theoretically with mean zero and variance one). Finally, the data $X_{\upsilon,\tau}$ (flows in the original domain) is obtained from

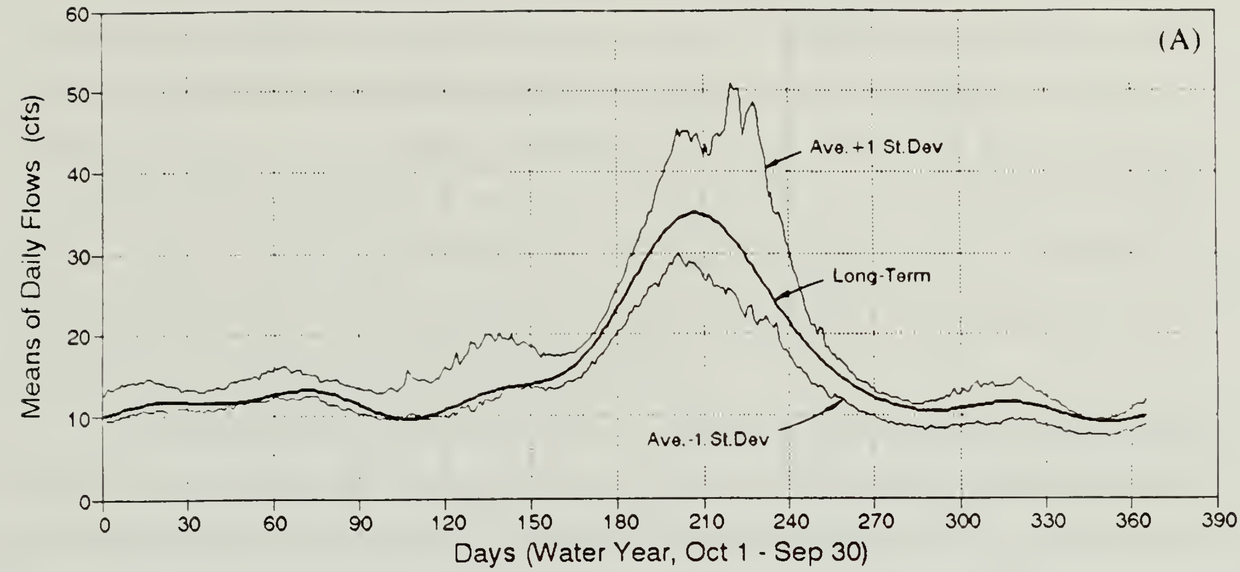
$$
X_{\upsilon,\tau} = \exp[\overline{Y}_{\tau} + s_{\tau}(y)\, Y_{\upsilon,\tau}] \qquad \text{Eq.(15)}
$$

where $\overline{Y}_{\tau}$ and $s_{\tau}(y)$ are the estimated long-term periodic mean and standard deviation of the logarithms of daily flows for the NFVR above Narrows Canyon. Equivalent periodic functions for the extended period [1926-1988], but in the original flow domain, were shown in Figure 6. A computer program was implemented for synthetic data generation.

6.2 Validation of Generated Data

The last step in any stochastic modeling effort relates to the validity of the chosen model. In order to evaluate the reliability of the fitted ARMA(4,2) to model daily-flows at site (9) some statistical analysis will be performed based on actual data generation as described in Section 6.1. Before concentrating in the analysis, it should be pointed out that stochastic models are built to "resemble" the main statistical characteristics of the historic series in the statistical sense. That is, it should not be expected that synthetic series based on the model will give "exactly" the same statistical characteristics as depicted by the historic series, but rather, we should look for a close resemblance to those characteristics.

Fifteen hydrological series of daily flows, 70 years long each (for a total of 1050 years), were generated. Basic statistics were computed and compiled separately for each hydrological series in the original flow domain. Plotted in Figure 10 are the long-term basic statistics, daily-means in Graph (A) and daily-standard deviation in Graph (B) (darker lines), together with the upper and lower confidence bands of the generated statistics. The



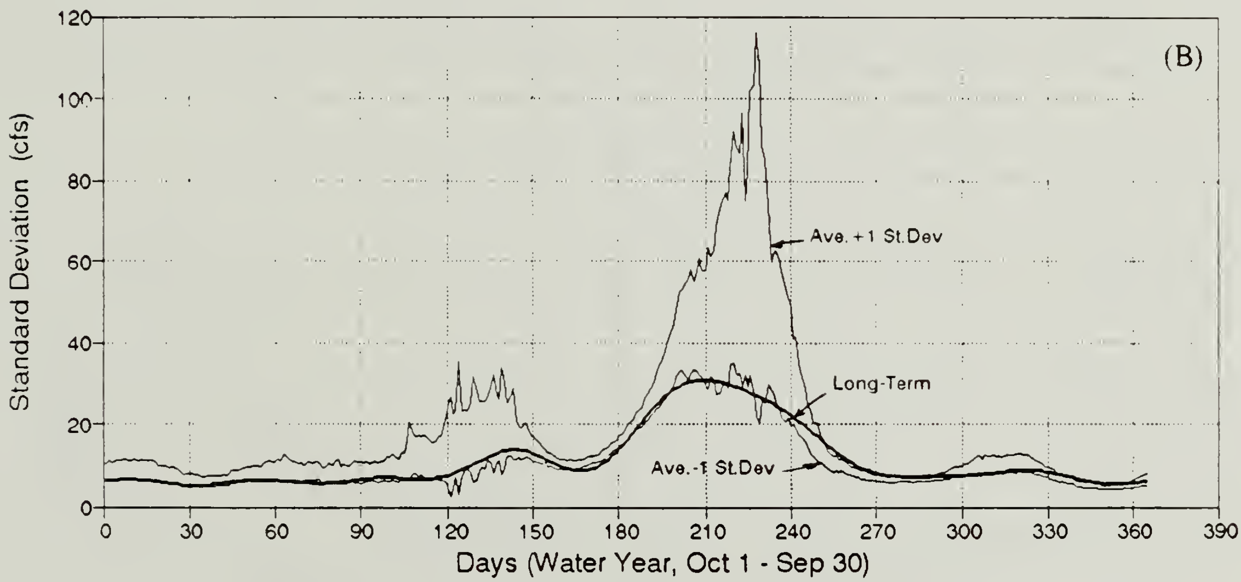


Fig.10 Historical Daily Means - Graph (A), and Standard Deviation - Graph (B), and Confidence Bands of Generated Statistics

confidence band is computed as positive and negative one-standard error relative to the average of the 15 series.

In general, the results show that long-term statistics are satisfactorily reproduced. Notice that the generated statistics follow very closely to the periodic pattern of the historic series. The confidence bands of the generated statistics enclose the historical statistics, although in the case of the standard deviation, the latter lies close to the lower bound during the snowmelt season. This is not unusual since the band is only one standard error above and below the generated mean. Presumably, the larger than expected flow variability of the synthetically generated series is due to the increase in flow variance introduced at site (9) during the flow extension procedure, see Section 4.2.

The generated first serial autocorrelation function $r_{1,\tau}$ (not shown) resulted slightly smoother than the historical one, since a constant (non-periodic) autocorrelation coefficient was adopted during the generation. Moreover, the average value of the $r_{1,\tau}$ function (for τ= 1 to 365) increased from 0.86 for the historical series (see Figure 8) to 0.92 for the synthetic series. The flow dependence structure of the synthetic series was also compared with the historic series by plotting the correlograms. Figure 11 displays the two corresponding correlograms, showing a very good reproduction of the correlation structure of flows.

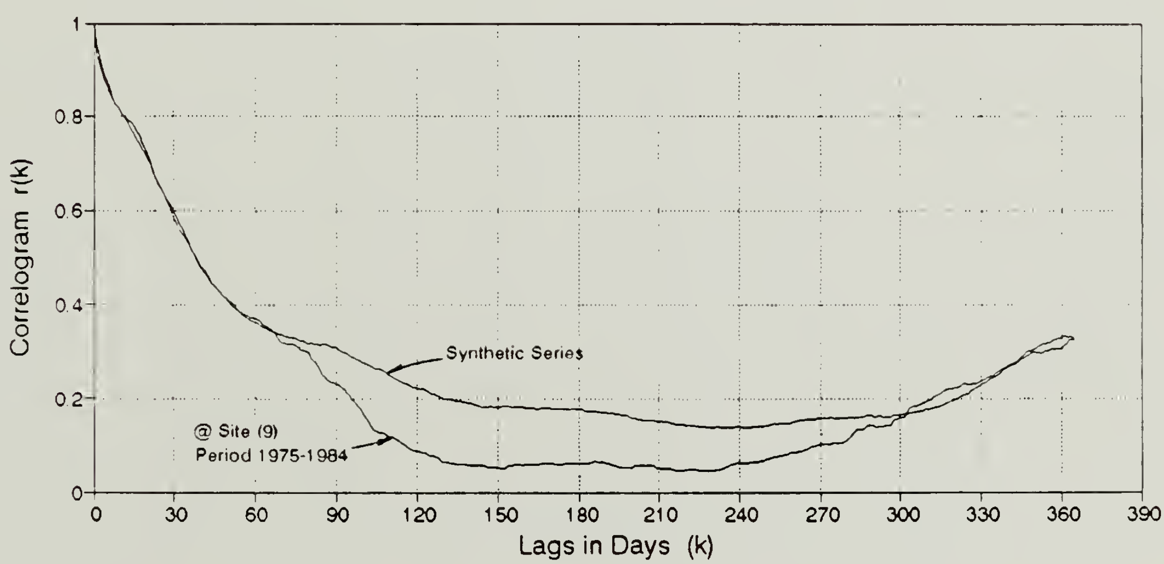


Fig.11 Periodic Correlogram of Historical and Synthetic Series of Daily Flows

This flow dependence aspect of the whole modeling effort is considered critical when judging the performance of the stochastic model. Particularly in this case, due to the extremely large persistence of flows in the upper region of the NFVR basin. Note that the correlogram shown in Figure 11 at site (9) is the same as the one shown in Figure 9 (darker line), except that in the case of Figure 9 the flow series was standardized prior to applying Eq.(6), and consequently, removing the periodicity embedded in the daily flow series.

In summary, it can be concluded that the resemblance of generated and historical statistics is as expected and desired, making the model useful for the generation of synthetic flows in the area of the NFVR above Zion Narrows. However, it is important to point out that the stochastic model developed in this study has the characteristic of being univariate, that is, the stochastic component of the model was developed using information from a single river site. While this is not a limitation for using the model as a tool to simulate surface runoff conditions at a local level, it prevents the analyst from using the generated flows in conjunction with other existing flow records in the area, in a more regional type of analysis. In case that several concurrent (in time) hydrologic series were necessary for the analysis, the multivariate stochastic analysis and multivariate modeling will become imperative.

7.0 LITERATURE CITED

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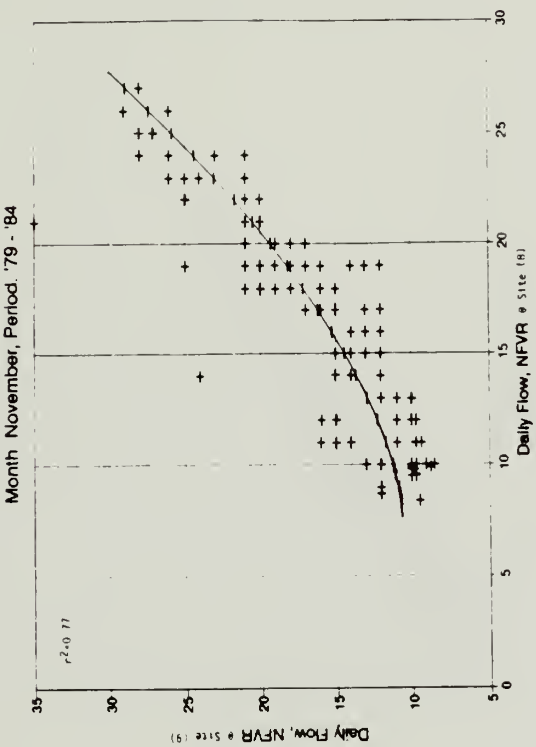
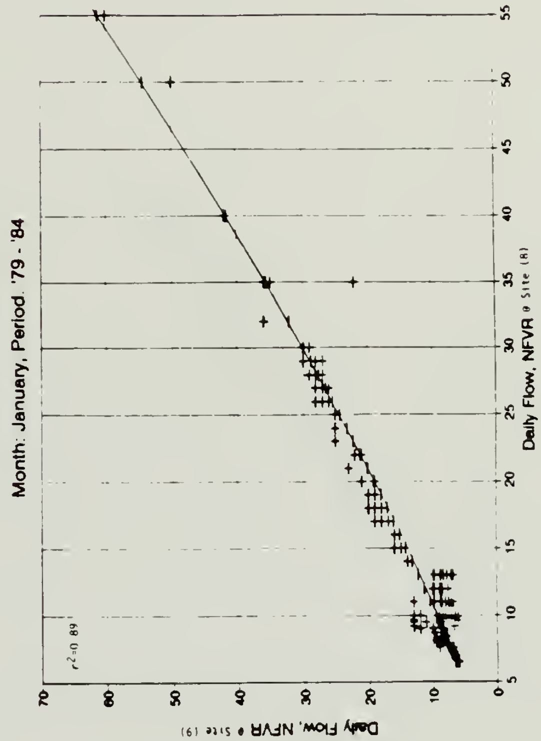
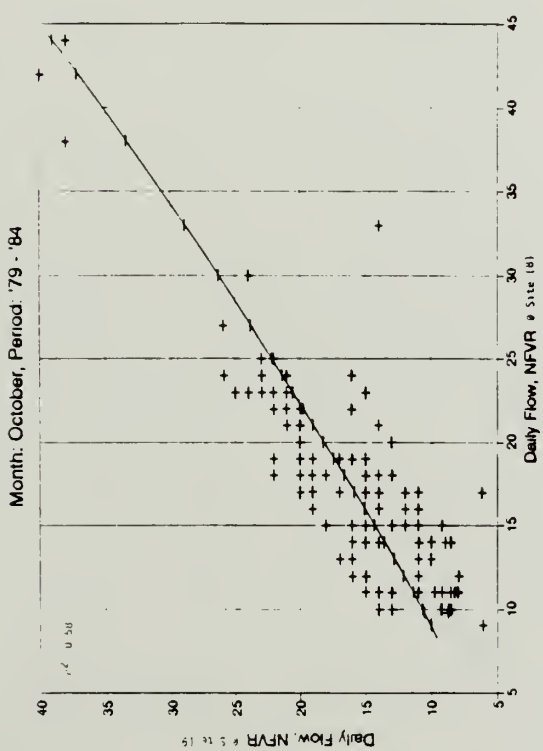
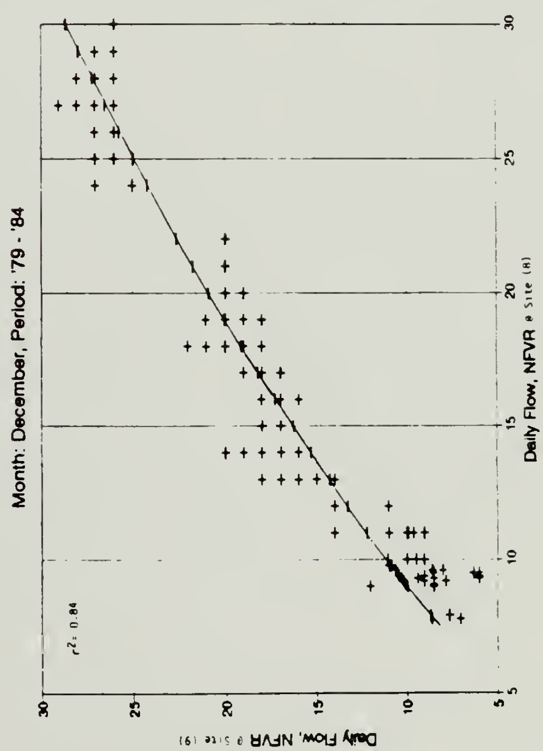
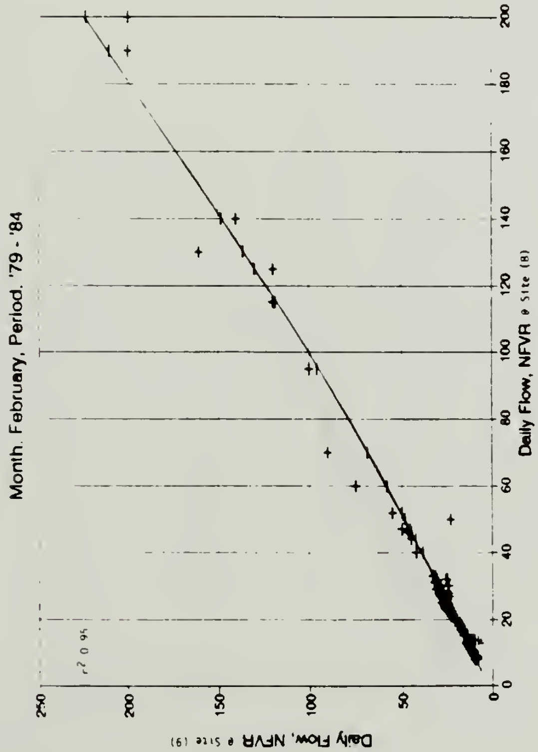
Appendix I. Regression of Mean-Daily Flows at Site (9) versus Mean-Daily Flows at Site (8)

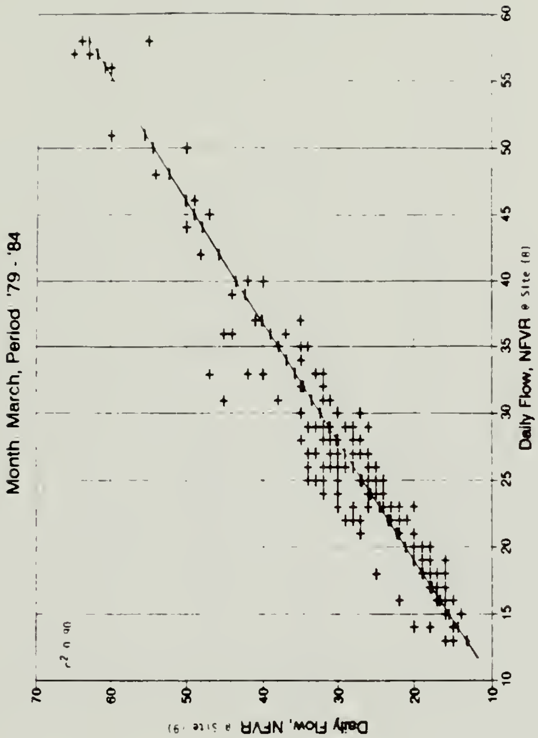


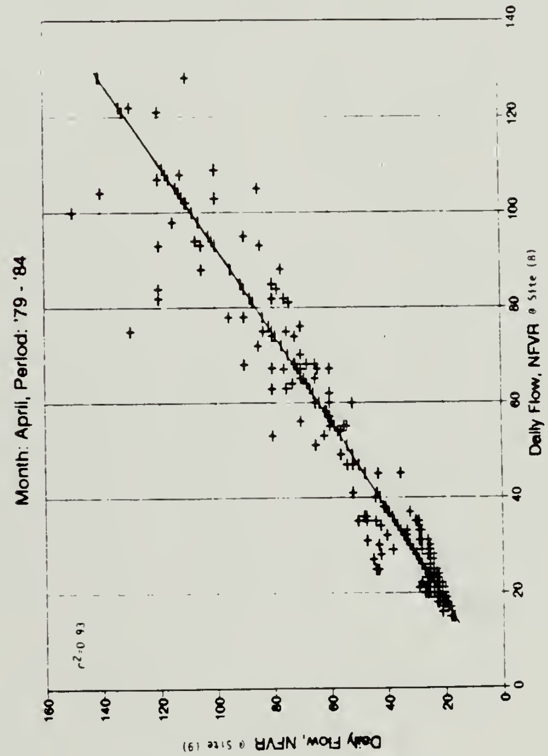


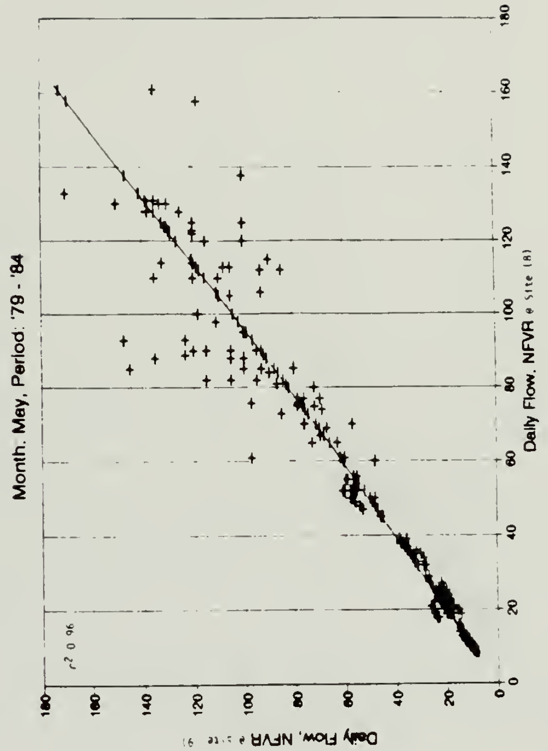


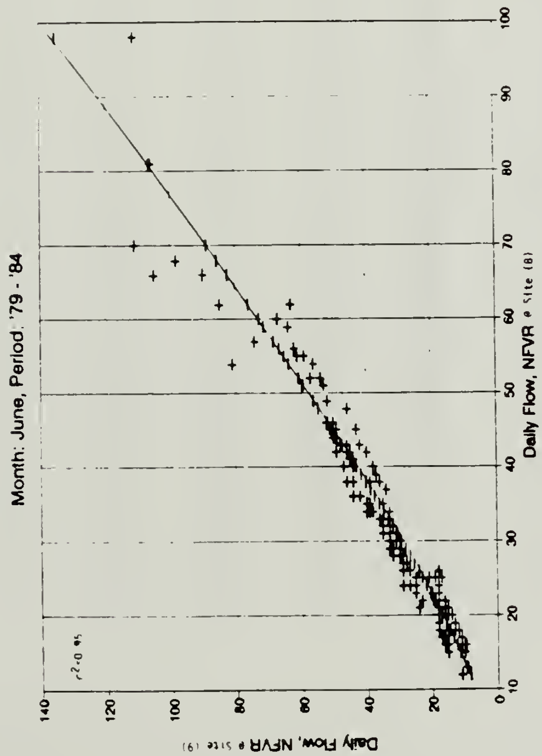


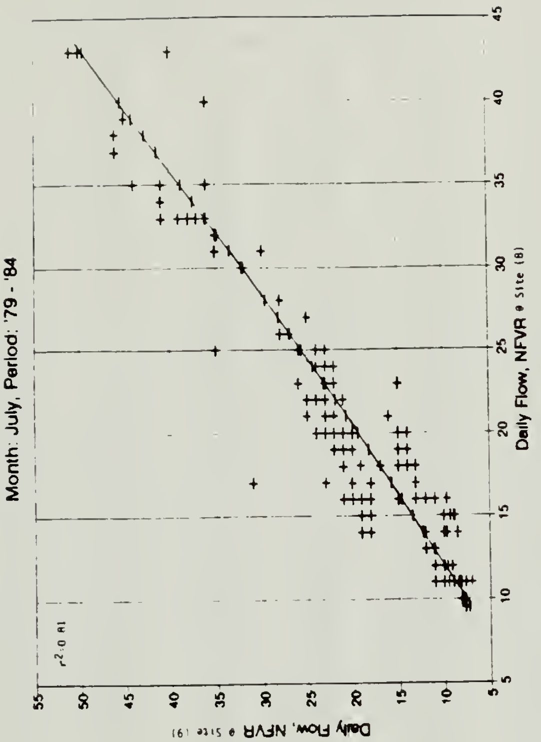

Month: February, Period: '79 - '84
Daily Flow, NFVR @ Site (8)
Daily Flow, NFVR @ Site (9)
r² 0.95

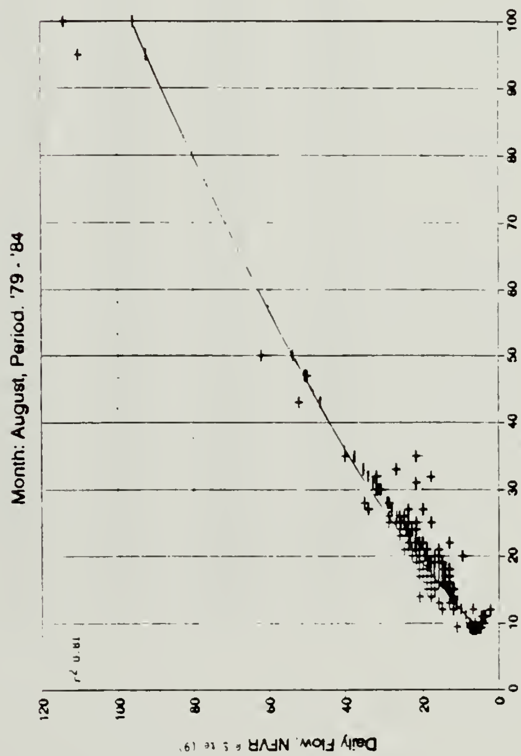

Month: March, Period '79 - '84
Daily Flow, NFVR @ Site (8)
Daily Flow, NFVR @ Site (9)
r² 0.90


Month: April, Period: '79 - '84
Daily Flow, NFVR @ Site (8)
Daily Flow, NFVR @ Site (9)
r²:0.93


Month: May, Period: '79 - '84
Daily Flow, NFVR @ Site (8)
Daily Flow, NFVR @ Site (9)
r² 0.96


Month: June, Period: '79 - '84
Daily Flow, NFVR @ Site (8)
Daily Flow, NFVR @ Site (9)


Month: July, Period: '79 - '84
Daily Flow, NFVR @ Site (8)
Daily Flow, NFVR @ Site (9)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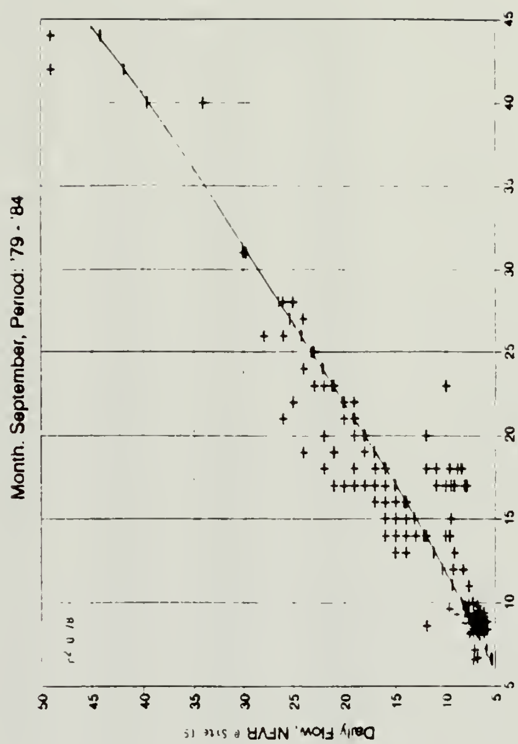
Daily Flow, NFVR @ Site (8)
Daily Flow, NFVR @ Site (9)


Month: September, Period: '79 - '84
Daily Flow, NFVR @ Site (8)
Daily Flow, NFVR @ Site (9)

Appendix II : Long-Term Estimates of Periodic Mean and Periodic Standard Deviation of Daily Flows for the NFVR above Narrows Canyon

Day No.	.. Estimated .. Mean	Std.Dev	... Fitted ... Mean	Std.Dev
1	11.72	10.05	10.13	6.43
2	12.65	10.21	10.24	6.51
3	10.20	5.62	10.35	6.58
4	10.60	5.85	10.46	6.64
5	10.32	5.43	10.58	6.69
6	10.96	4.20	10.69	6.73
7	11.14	5.18	10.80	6.76
8	9.25	4.46	10.91	6.78
9	10.69	5.90	11.01	6.78
10	10.03	5.47	11.11	6.77
11	11.05	5.49	11.20	6.74
12	9.27	6.61	11.29	6.70
13	11.15	6.39	11.37	6.65
14	11.55	6.33	11.45	6.59
15	10.90	6.18	11.52	6.51
16	11.66	6.55	11.58	6.43
17	12.17	7.07	11.63	6.34
18	10.99	6.20	11.68	6.24
19	12.63	6.63	11.72	6.14
20	12.23	6.59	11.75	6.03
21	11.04	6.75	11.77	5.93
22	10.59	6.22	11.79	5.83
23	11.35	5.58	11.80	5.73
24	11.01	6.15	11.80	5.63
25	11.09	5.58	11.80	5.54
26	12.82	6.75	11.80	5.47
27	12.23	5.11	11.79	5.40
28	11.52	4.90	11.77	5.34
29	11.89	5.19	11.75	5.29
30	12.21	5.90	11.73	5.26
31	11.35	5.43	11.71	5.24
32	12.05	5.08	11.69	5.23
33	11.21	6.48	11.67	5.24
34	11.75	5.36	11.64	5.27
35	12.72	5.66	11.62	5.30
36	11.08	4.76	11.61	5.35
37	11.77	5.03	11.59	5.41
38	11.68	5.34	11.58	5.48
39	12.28	5.18	11.57	5.56
40	11.87	5.07	11.57	5.64
41	11.47	4.68	11.57	5.74
42	11.65	7.11	11.58	5.83
43	11.49	5.89	11.59	5.93
44	12.32	6.72	11.61	6.03
45	13.01	6.65	11.64	6.13
46	12.00	5.56	11.67	6.22
47	12.87	6.45	11.71	6.31
48	12.84	6.26	11.76	6.39
49	12.31	6.18	11.81	6.46
50	11.96	6.69	11.87	6.53
51	11.59	5.96	11.93	6.58
52	12.30	6.79	12.00	6.62
53	11.03	6.25	12.08	6.65
54	12.06	6.68	12.15	6.67
55	12.53	7.09	12.23	6.67
56	12.37	6.70	12.31	6.67
57	11.79	6.29	12.40	6.65
58	11.97	6.40	12.48	6.62
59	11.18	6.32	12.57	6.58
60	13.18	6.42	12.65	6.53

Day No.	.. Estimated .. Mean	Std.Dev	... Fitted .... Mean	Std.Dev
61	11.45	6.52	12.73	6.47
62	13.29	6.05	12.81	6.40
63	12.70	5.51	12.88	6.33
64	13.30	6.42	12.95	6.26
65	12.04	6.11	13.01	6.19
66	12.17	5.05	13.07	6.11
67	11.29	5.90	13.11	6.04
68	12.23	5.69	13.15	5.97
69	13.94	6.28	13.18	5.91
70	13.66	6.94	13.20	5.85
71	14.74	7.24	13.21	5.80
72	12.57	5.20	13.21	5.77
73	13.19	6.20	13.20	5.74
74	13.15	6.16	13.18	5.72
75	12.48	5.87	13.14	5.72
76	13.30	6.02	13.09	5.73
77	12.67	5.58	13.03	5.75
78	11.71	5.71	12.96	5.78
79	13.44	6.81	12.88	5.83
80	12.96	6.53	12.79	5.88
81	12.57	6.13	12.69	5.95
82	12.30	5.48	12.58	6.02
83	14.11	6.85	12.46	6.11
84	13.43	6.39	12.33	6.19
85	12.13	5.98	12.19	6.29
86	13.71	6.66	12.05	6.38
87	12.74	6.70	11.90	6.48
88	12.81	6.72	11.75	6.57
89	11.68	6.70	11.60	6.66
90	12.98	7.72	11.45	6.75
91	11.98	6.17	11.29	6.83
92	12.50	6.52	11.14	6.91
93	8.62	5.59	10.98	6.97
94	8.61	5.76	10.84	7.02
95	9.85	6.51	10.69	7.07
96	8.57	5.95	10.56	7.10
97	8.99	6.13	10.43	7.12
98	9.98	6.26	10.30	7.13
99	9.33	5.82	10.19	7.13
100	10.37	6.68	10.09	7.12
101	9.39	6.76	10.00	7.10
102	9.94	6.31	9.92	7.07
103	9.94	5.68	9.85	7.03
104	9.82	6.79	9.80	6.99
105	10.95	7.72	9.76	6.95
106	10.14	10.77	9.74	6.91
107	10.05	7.91	9.73	6.87
108	9.17	6.65	9.73	6.84
109	9.11	6.44	9.75	6.81
110	10.84	10.01	9.78	6.80
111	11.61	9.05	9.83	6.79
112	11.21	7.78	9.89	6.80
113	10.39	7.96	9.96	6.83
114	10.18	7.13	10.05	6.88
115	10.19	6.60	10.14	6.96
116	10.37	7.06	10.25	7.05
117	10.51	6.89	10.37	7.17
118	11.39	6.78	10.49	7.31
119	12.93	7.97	10.63	7.49
120	10.76	6.93	10.77	7.68

(continuation)

Day No.	.. Estimated .. Mean	Std.Dev	... Fitted ... Mean	Std.Dev
121	12.60	12.16	10.91	7.90
122	10.86	9.51	11.06	8.15
123	11.38	9.32	11.22	8.42
124	10.77	8.72	11.37	8.71
125	10.90	8.04	11.53	9.03
126	10.52	7.37	11.69	9.35
127	10.79	7.62	11.84	9.70
128	10.90	7.49	11.99	10.05
129	11.00	7.19	12.14	10.41
130	10.09	6.23	12.29	10.77
131	11.47	7.59	12.42	11.13
132	11.85	6.21	12.56	11.48
133	10.87	5.42	12.68	11.83
134	12.15	6.50	12.80	12.16
135	12.62	6.99	12.91	12.47
136	12.57	6.79	13.02	12.76
137	14.11	27.22	13.12	13.02
138	14.74	25.27	13.20	13.26
139	12.91	17.24	13.29	13.46
140	12.86	18.95	13.36	13.62
141	13.57	21.72	13.43	13.75
142	14.33	21.33	13.49	13.83
143	13.93	26.00	13.55	13.87
144	13.73	16.44	13.60	13.87
145	13.09	13.44	13.65	13.83
146	14.98	11.87	13.70	13.75
147	13.66	10.65	13.75	13.62
148	15.00	10.18	13.80	13.46
149	13.22	8.03	13.85	13.26
150	11.67	7.96	13.90	13.02
151	12.68	8.12	13.96	12.76
152	15.97	9.12	14.03	12.47
153	15.06	8.24	14.10	12.16
154	14.22	10.34	14.18	11.83
155	15.74	9.54	14.28	11.49
156	13.92	8.10	14.39	11.15
157	16.28	12.45	14.51	10.81
158	13.20	8.97	14.65	10.48
159	15.84	8.89	14.81	10.16
160	15.16	8.21	14.99	9.85
161	15.89	9.60	15.19	9.58
162	14.86	8.69	15.40	9.33
163	18.83	11.99	15.65	9.12
164	15.81	9.81	15.91	8.96
165	15.95	11.51	16.20	8.84
166	17.86	10.75	16.51	8.77
167	16.72	9.35	16.85	8.76
168	17.71	9.43	17.21	8.80
169	17.29	8.97	17.60	8.91
170	16.63	8.11	18.01	9.08
171	20.13	11.21	18.44	9.31
172	16.72	8.62	18.90	9.60
173	20.94	12.38	19.38	9.96
174	20.26	11.22	19.88	10.38
175	21.41	11.57	20.41	10.86
176	21.87	12.87	20.95	11.40
177	20.57	12.97	21.51	11.99
178	23.12	15.75	22.08	12.64
179	23.40	15.81	22.67	13.33
180	27.38	20.55	23.27	14.07
181	24.18	18.39	23.87	14.84
182	25.59	17.37	24.49	15.64
183	20.92	12.77	25.11	16.47
184	22.14	13.59	25.73	17.32
185	19.53	11.93	26.35	18.18
186	24.98	14.97	26.97	19.05
187	22.77	16.68	27.58	19.92
188	24.35	18.93	28.18	20.79
189	30.44	24.09	28.77	21.64
190	30.16	27.24	29.35	22.48
191	25.87	21.91	29.91	23.30
192	27.39	20.58	30.45	24.09
193	28.21	18.86	30.97	24.84
194	31.83	20.02	31.46	25.57
195	35.53	22.88	31.93	26.25
196	27.99	20.86	32.37	26.89
197	28.12	22.68	32.78	27.49
198	35.40	31.73	33.15	28.03
199	37.83	35.59	33.49	28.54
200	32.52	27.91	33.80	28.99
201	38.06	30.99	34.07	29.39
202	36.71	29.30	34.30	29.74
203	40.68	39.66	34.49	30.04
204	38.83	37.24	34.65	30.29
205	43.52	39.83	34.76	30.49
206	45.38	33.57	34.83	30.65
207	36.72	26.70	34.86	30.77
208	33.73	26.68	34.85	30.84
209	42.61	39.52	34.80	30.88
210	34.57	31.05	34.72	30.88
211	45.79	39.99	34.59	30.84
212	33.24	27.96	34.42	30.78
213	26.95	22.40	34.22	30.68
214	27.07	21.87	33.98	30.56
215	27.05	22.74	33.71	30.42
216	27.16	24.18	33.41	30.25
217	28.77	31.87	33.07	30.07
218	30.50	32.25	32.71	29.87
219	30.38	29.85	32.32	29.65
220	32.86	33.48	31.90	29.42
221	28.30	29.64	31.47	29.18
222	32.62	34.28	31.01	28.93
223	27.91	26.86	30.53	28.66
224	27.91	26.55	30.03	28.38
225	31.44	30.02	29.52	28.10
226	27.58	26.22	29.00	27.80
227	28.04	25.88	28.47	27.49
228	27.58	26.77	27.93	27.18
229	26.94	26.10	27.39	26.85
230	28.55	27.67	26.84	26.51
231	27.05	27.31	26.29	26.15
232	25.93	24.47	25.74	25.78
233	22.67	24.28	25.19	25.40
234	26.64	28.28	24.64	25.01
235	23.05	23.28	24.10	24.60
236	21.79	21.72	23.57	24.17
237	20.98	21.36	23.04	23.73
238	20.41	21.41	22.52	23.27
239	21.37	21.61	22.02	22.79
240	18.89	20.39	21.52	22.30

(continuation)

Day No.	.. Estimated .. Mean	Std.Dev	... Fitted ... Mean	Std.Dev
241	18.20	18.93	21.03	21.80
242	18.13	19.53	20.56	21.28
243	18.21	18.31	20.10	20.74
244	24.72	25.29	19.65	20.20
245	26.62	26.32	19.22	19.64
246	20.99	21.58	18.80	19.07
247	21.73	22.82	18.39	18.50
248	21.14	20.45	18.00	17.92
249	19.94	19.39	17.62	17.34
250	20.02	18.90	17.25	16.76
251	20.57	18.71	16.90	16.18
252	20.11	17.50	16.56	15.60
253	18.90	14.80	16.24	15.03
254	18.03	13.87	15.92	14.47
255	16.62	13.44	15.62	13.92
256	14.96	12.51	15.33	13.39
257	14.22	11.30	15.06	12.87
258	13.44	10.13	14.79	12.37
259	14.78	11.10	14.53	11.88
260	13.06	9.99	14.29	11.42
261	11.18	8.45	14.05	10.99
262	11.33	8.12	13.82	10.58
263	13.36	9.61	13.60	10.19
264	9.74	6.82	13.39	9.83
265	11.47	8.17	13.19	9.50
266	13.46	9.40	13.00	9.19
267	10.67	7.86	12.81	8.92
268	11.17	7.81	12.63	8.66
269	12.20	8.27	12.46	8.44
270	10.39	7.80	12.30	8.24
271	9.25	6.67	12.14	8.06
272	9.56	7.07	11.99	7.91
273	9.42	6.70	11.85	7.78
274	11.37	8.61	11.72	7.67
275	10.98	8.52	11.59	7.57
276	12.93	8.73	11.47	7.50
277	12.92	9.06	11.36	7.44
278	9.90	6.70	11.25	7.39
279	11.53	7.60	11.16	7.36
280	12.25	8.71	11.07	7.33
281	10.27	7.90	10.99	7.32
282	13.72	10.20	10.91	7.31
283	11.96	9.44	10.85	7.31
284	12.67	10.33	10.79	7.31
285	13.17	10.58	10.74	7.32
286	13.01	10.72	10.71	7.33
287	10.31	8.79	10.67	7.34
288	8.92	6.95	10.65	7.35
289	13.23	8.07	10.64	7.36
290	12.07	7.09	10.63	7.38
291	10.56	5.53	10.63	7.39
292	10.21	5.37	10.64	7.41
293	10.11	5.65	10.66	7.43
294	9.93	7.02	10.68	7.44
295	9.41	7.40	10.71	7.46
296	9.16	5.69	10.74	7.49
297	9.88	6.01	10.79	7.51
298	10.12	5.77	10.83	7.54
299	11.81	6.62	10.88	7.57
300	9.60	5.30	10.93	7.61

Day No.	.. Estimated .. Mean	Std.Dev	... Fitted .... Mean	Std.Dev
301	11.25	6.67	10.99	7.65
302	12.41	6.99	11.05	7.69
303	9.77	5.85	11.11	7.74
304	10.01	6.04	11.17	7.80
305	11.82	7.52	11.22	7.85
306	13.24	8.37	11.28	7.92
307	14.10	8.99	11.34	7.99
308	12.58	7.64	11.39	8.06
309	12.67	8.21	11.44	8.13
310	11.06	10.43	11.49	8.21
311	10.42	7.00	11.53	8.29
312	12.07	9.04	11.57	8.37
313	11.06	7.62	11.60	8.44
314	10.73	10.16	11.62	8.52
315	12.28	8.10	11.63	8.59
316	16.17	12.44	11.64	8.66
317	11.43	9.25	11.64	8.72
318	9.37	6.45	11.63	8.77
319	10.04	10.07	11.62	8.82
320	10.51	8.72	11.59	8.85
321	12.45	17.20	11.56	8.87
322	11.69	16.28	11.52	8.88
323	10.70	7.38	11.47	8.88
324	9.40	5.87	11.41	8.86
325	12.63	8.25	11.34	8.83
326	9.61	6.30	11.27	8.79
327	10.62	6.99	11.19	8.72
328	12.35	8.98	11.10	8.65
329	10.71	9.02	11.01	8.56
330	10.63	7.63	10.91	8.45
331	9.74	7.38	10.81	8.33
332	10.28	7.08	10.70	8.20
333	10.10	7.01	10.59	8.06
334	9.35	6.45	10.48	7.91
335	9.73	7.69	10.37	7.75
336	10.75	6.97	10.26	7.58
337	10.13	6.75	10.15	7.41
338	8.82	5.41	10.04	7.24
339	9.05	5.29	9.94	7.07
340	9.14	5.49	9.84	6.90
341	10.64	6.03	9.74	6.73
342	10.92	7.23	9.65	6.57
343	8.96	7.16	9.57	6.41
344	13.34	9.77	9.49	6.27
345	8.73	8.74	9.42	6.13
346	8.43	5.08	9.36	6.01
347	10.32	5.73	9.31	5.90
348	10.57	6.77	9.26	5.81
349	8.60	5.54	9.23	5.73
350	9.42	5.38	9.21	5.67
351	8.00	4.76	9.20	5.63
352	10.08	5.38	9.20	5.60
353	11.40	6.05	9.20	5.59
354	9.42	5.36	9.22	5.59
355	10.23	5.52	9.25	5.61
356	9.04	5.22	9.29	5.65
357	9.13	5.50	9.34	5.69
358	9.67	6.28	9.40	5.75
359	10.41	6.90	9.47	5.82
360	8.42	4.46	9.55	5.90

(continuation)

Day No.	.. Estimated .. Mean	Std.Dev	... Fitted ... Mean	Std.Dev
361	8.43	5.11	9.63	5.98
362	9.42	9.21	9.72	6.07
363	10.51	6.74	9.81	6.16
364	10.26	6.88	9.91	6.25
365	8.50	6.91	10.02	6.34
Ave.->	15.26	14.45	15.26	14.45